Speed of Sound In QCD Plasma

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Introduction

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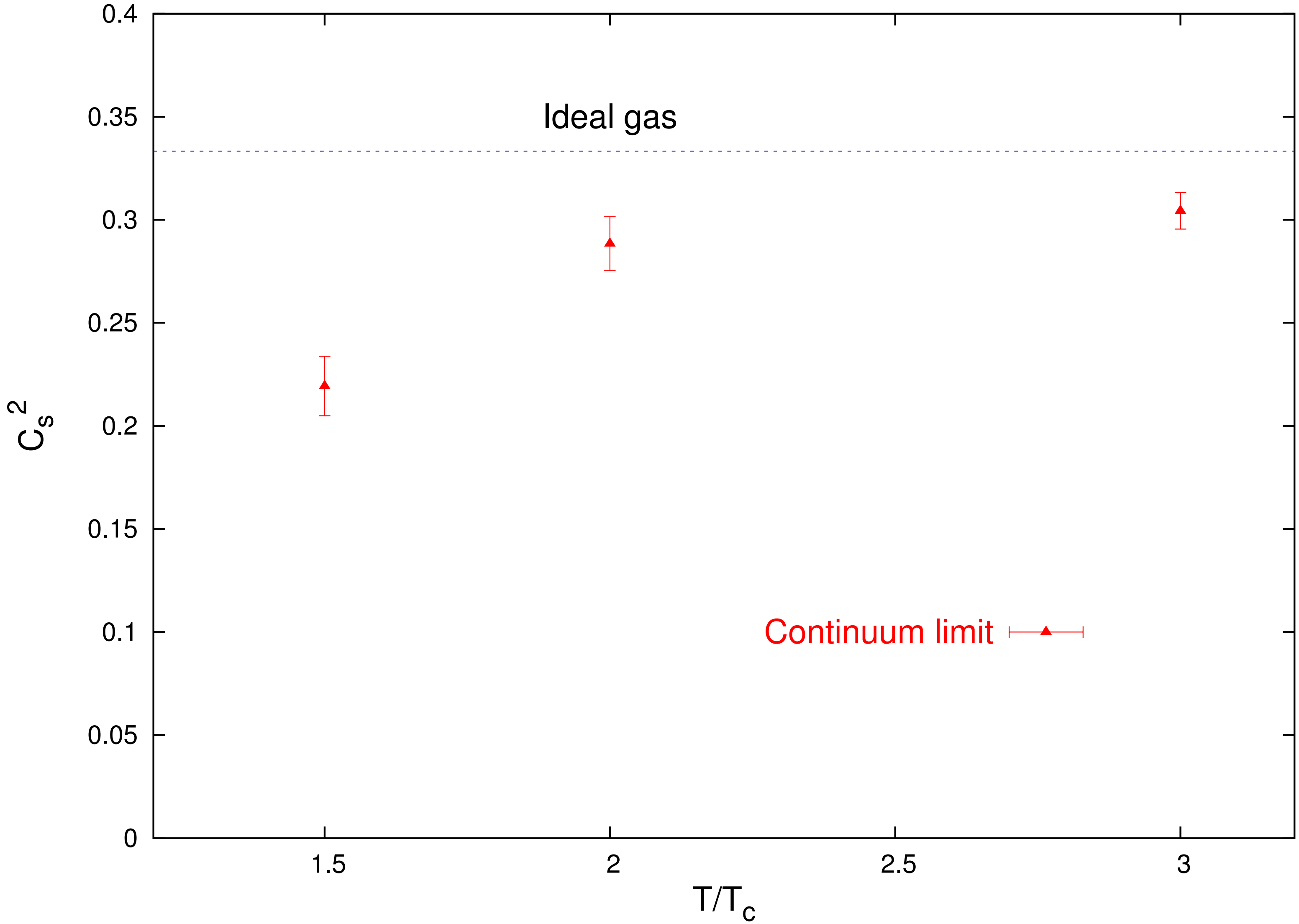
- These sound waves in a QCD plasma near equilibrium may affect its particle spectrum and can be observed.

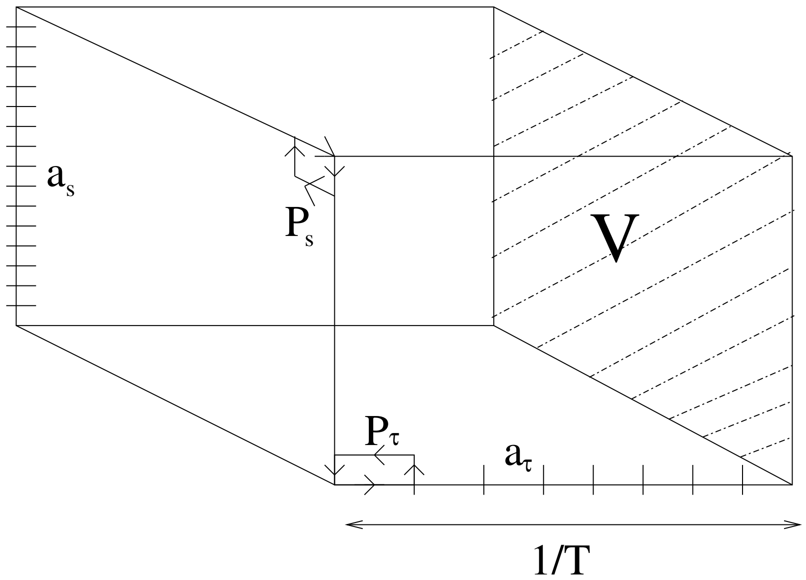
Speed of sound



Continuum limit of speed of sound in plasma phase of quenched QCD theory.

Action

- Wilson action on an asymmetric lattice :



$$S(U) = 2N_c K_s P_s + 2N_c K_\tau P_\tau$$

$$K_s \equiv \frac{1}{\xi g_s^2} \qquad K_\tau \equiv \frac{\xi}{g_\tau^2} \qquad \xi = \frac{a_s}{a_\tau}$$

Thermodynamics

- Partition function :

$$\mathcal{Z} = \int \mathcal{D}U e^{-S(U)}$$

- Energy density :

$$E = \frac{T^2}{V} \frac{\partial \ln \mathcal{Z}}{\partial T}\Big|_V$$

- Pressure :

$$P = T \frac{\partial \ln \mathcal{Z}}{\partial V}\Big|_T$$

Energy density

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$$\frac{E}{T^4} = 6 N_c N_\tau^4 \left[\frac{\Delta_s - \Delta_\tau}{g^2} - \left(c_s' \Delta_s + c_\tau' \Delta_\tau \right) \right]$$

where :

$$\Delta_j = \langle \bar{P}_j \rangle - \langle \bar{P}_0 \rangle$$

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- Karsch coefficients, evaluated upto one loop order :

$$g_i^{-2}(a_s, \xi) = g^{-2}(a) + c_i(\xi) + O[g^2(a)]$$

Specific heat

- Specific heat at constant volume :

$$\frac{C_v}{T^3} = \frac{1}{T^3}\frac{\partial E}{\partial T} =$$

$$\frac{4E}{T^4} - 6N_cN_\tau^4\Big[2g^{-2}\Delta_\tau + 4c'_\tau\Delta_\tau + (c''_s\Delta_s + c''_\tau\Delta_\tau)\Big] \quad +$$

$$36N_c^2N_s^3N_\tau^5\Big[g^{-4}var(\Delta_s - \Delta_\tau) \quad +$$

$$g^{-2}var(c_s{}'\Delta_s + c'_\tau\Delta_\tau, \Delta_s - \Delta_\tau) + var(c'_s\Delta_s + c'_\tau\Delta_\tau)\Big]$$

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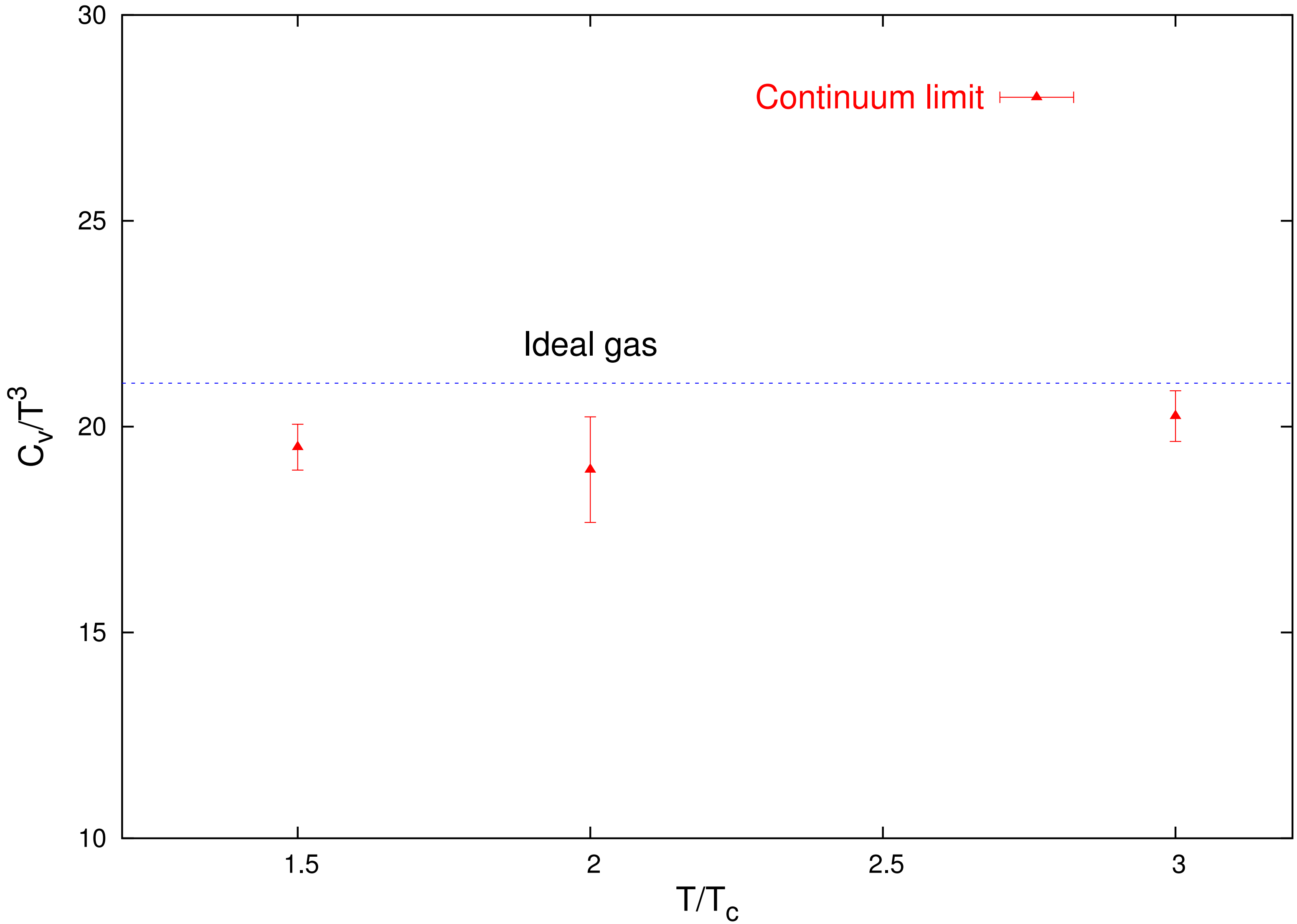
- $$c''_s(\xi = 1) = -0.298192$$
  $$c''_\tau(\xi = 1) = 0.333674$$

C_v Results



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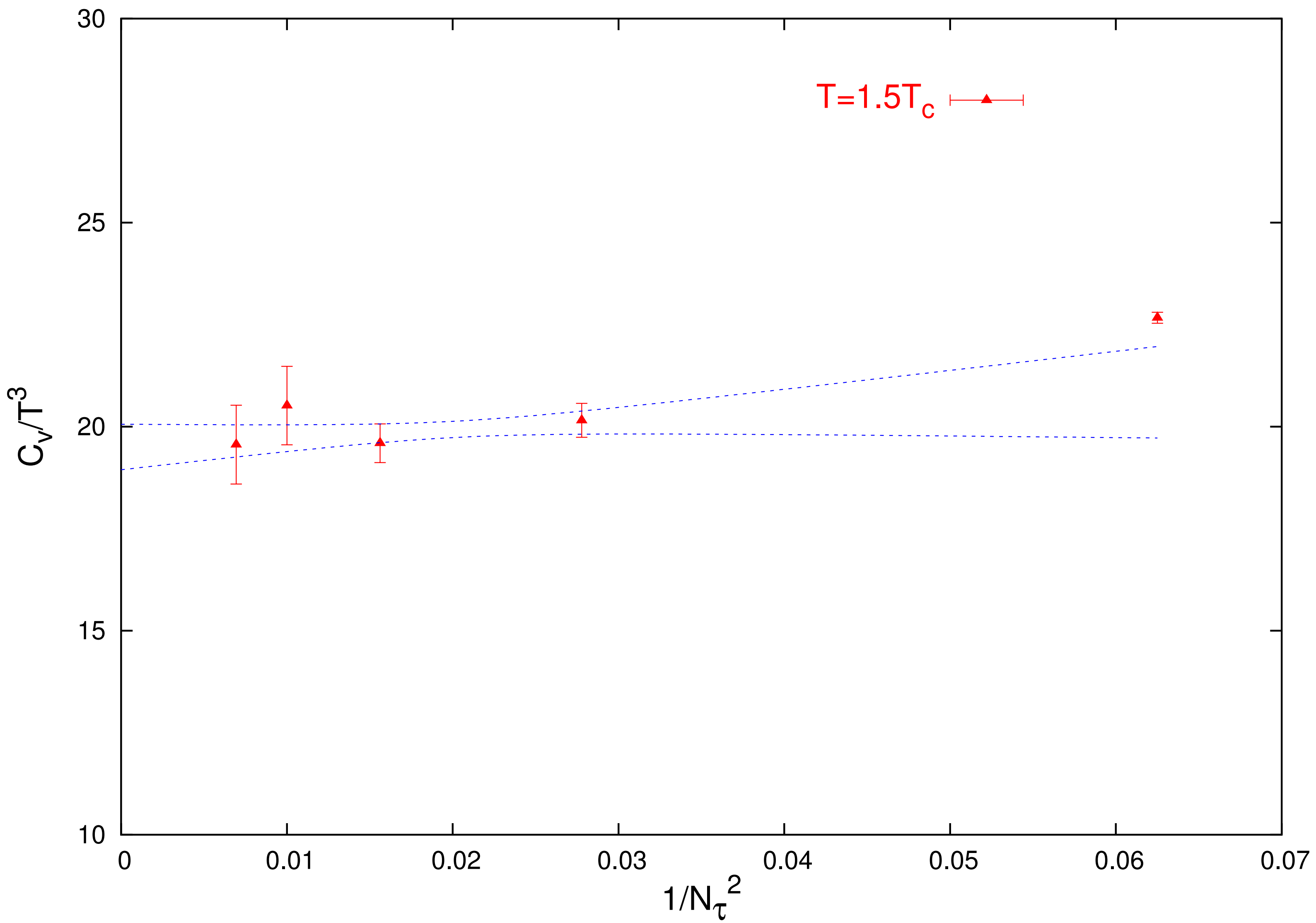
C_v Results



Speed of sound

- Speed of sound :

$$C_s^2 = \frac{\partial P}{\partial E}\Big|_s = \frac{1}{3} - \frac{1}{3} \cdot \frac{\frac{1}{T^3}\left(\frac{\partial D}{\partial T}\right)_V}{\frac{1}{T^3}\left(\frac{\partial E}{\partial T}\right)_V}$$

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- Interaction measure :

$$\frac{D}{T^4} = \frac{E - 3P}{T^4} = 6 N_c N_\tau^4 \left(a \frac{\partial g^{-2}}{\partial a}\right)(\Delta_s + \Delta_\tau)$$

Speed of sound

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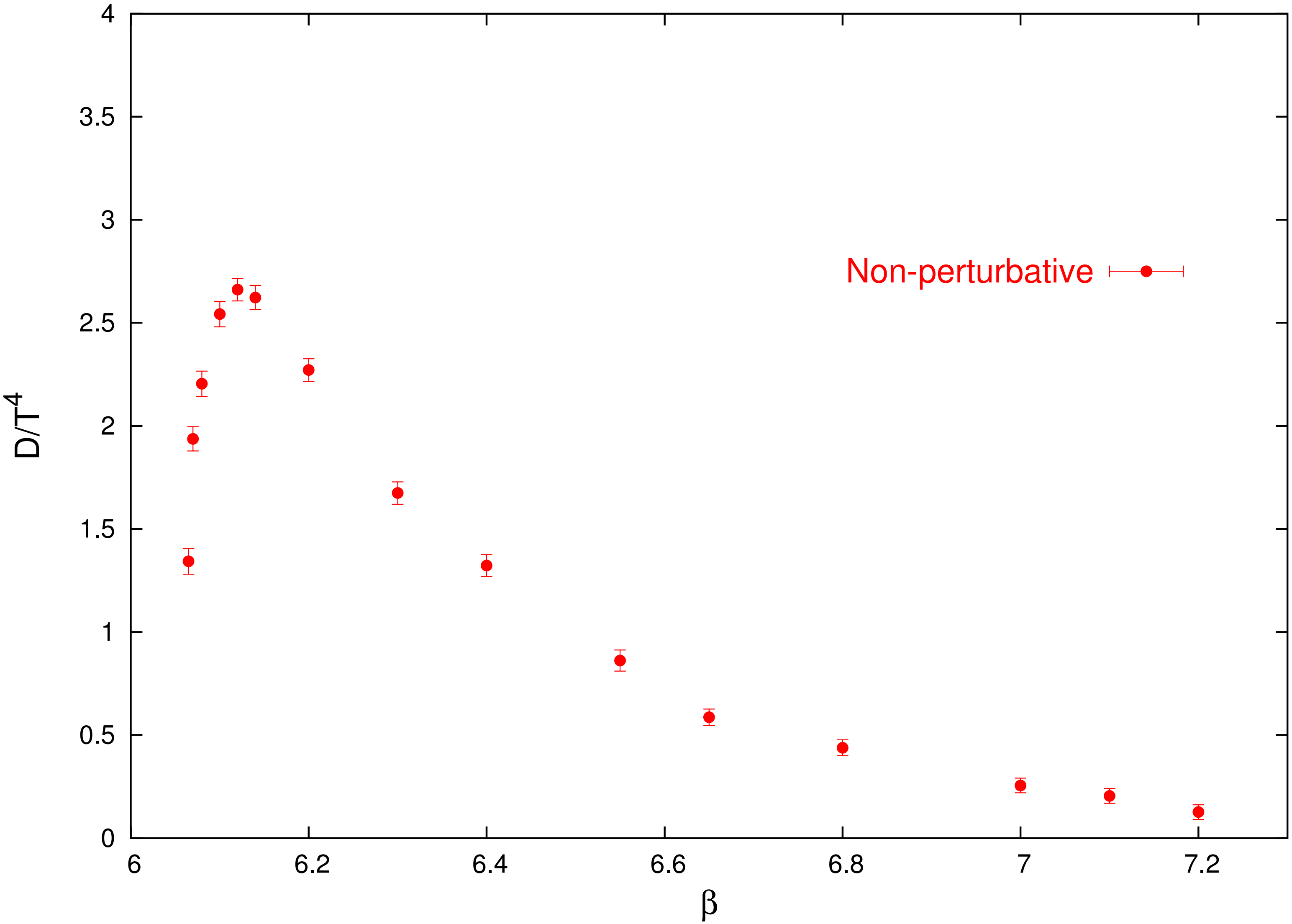
- Derivative of interaction measure :

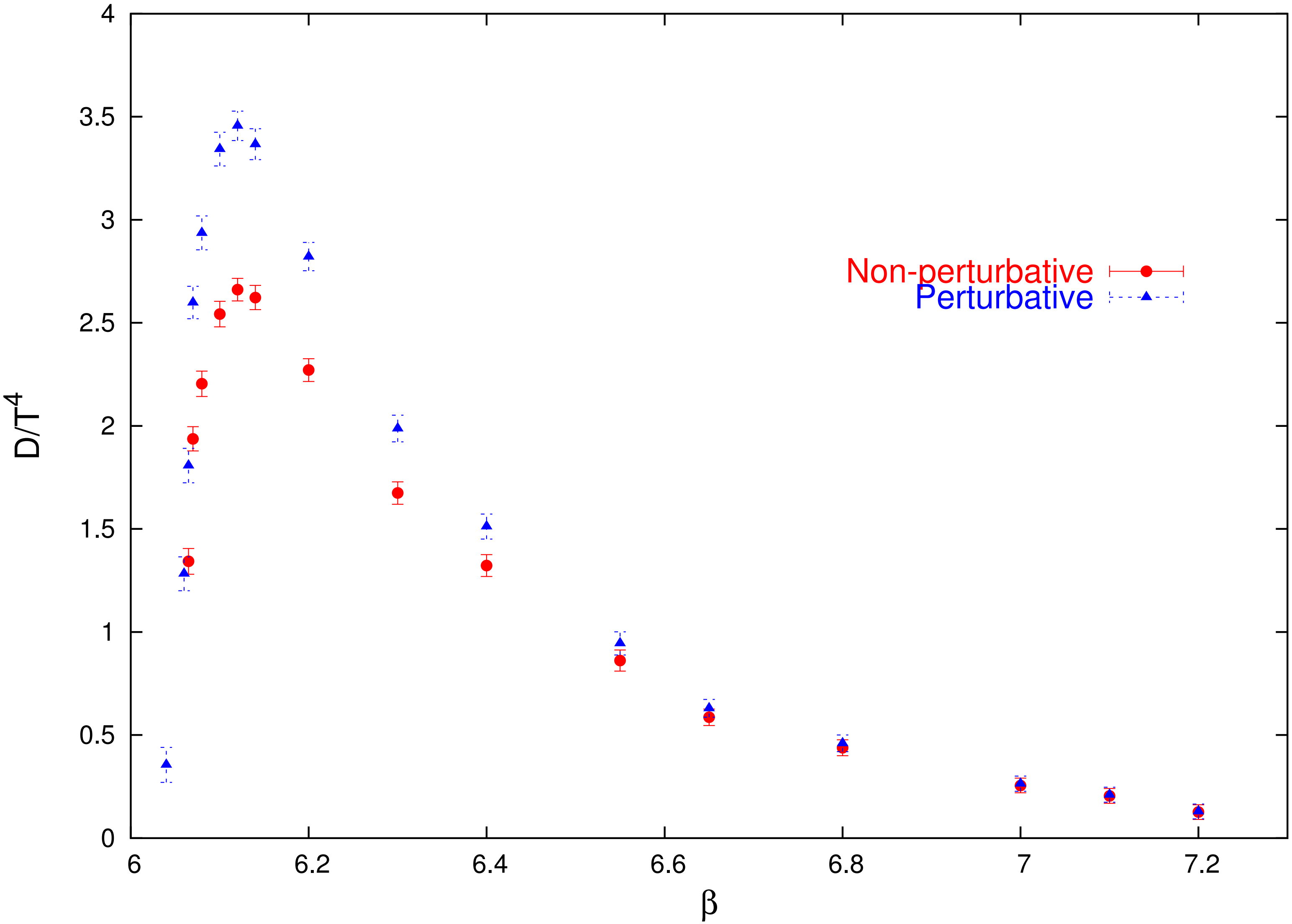
$$\frac{1}{T^3}\frac{\partial D}{\partial T} = \frac{4D}{T^4} - 12 N_c N_\tau^4 \left(a \frac{\partial g^{-2}}{\partial a}\right) \Delta_\tau$$

Lattice sizes



G.Boyd *et al.*, Nucl. Phys., B 469 (1996) 419.

Lattice sizes



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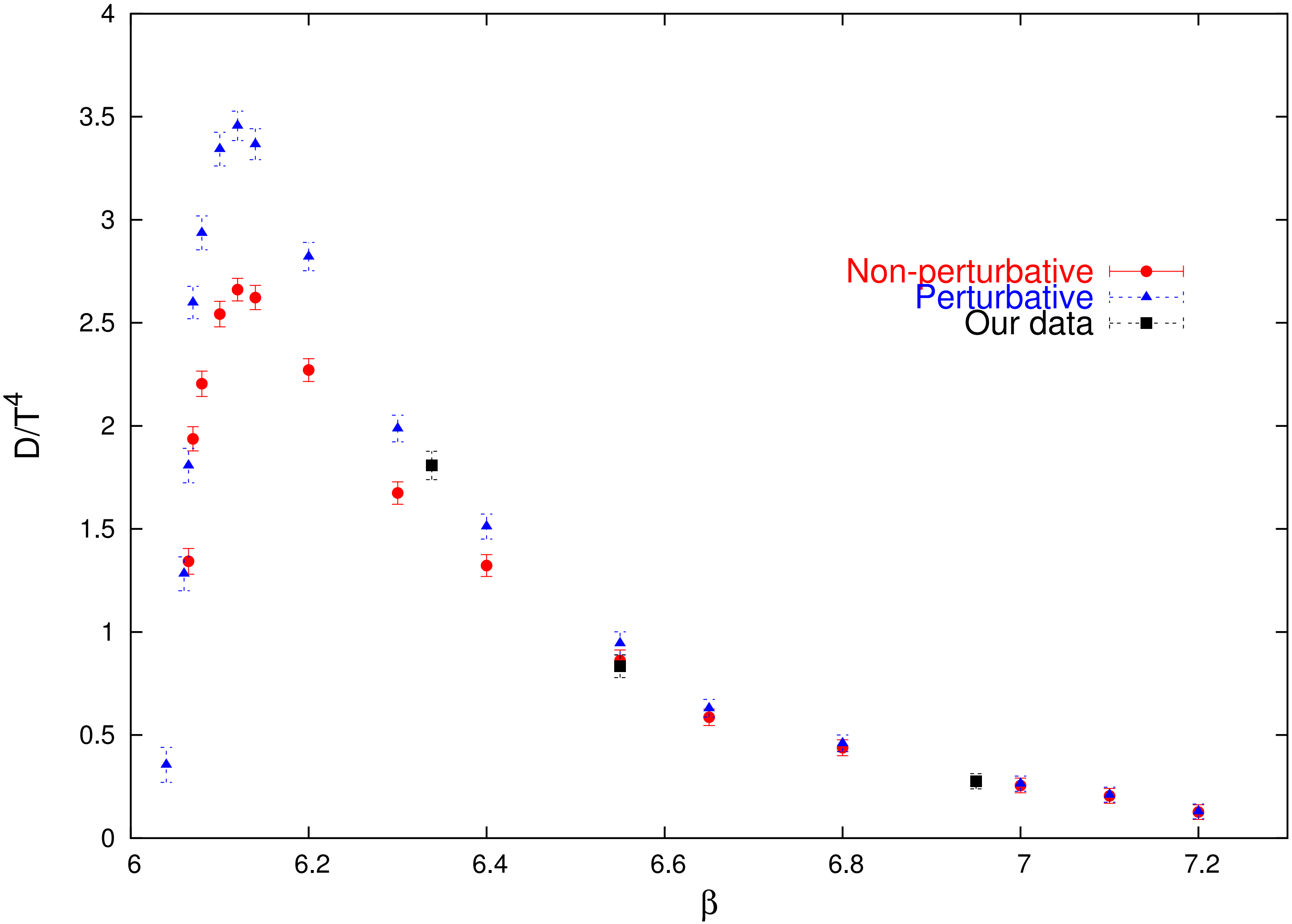

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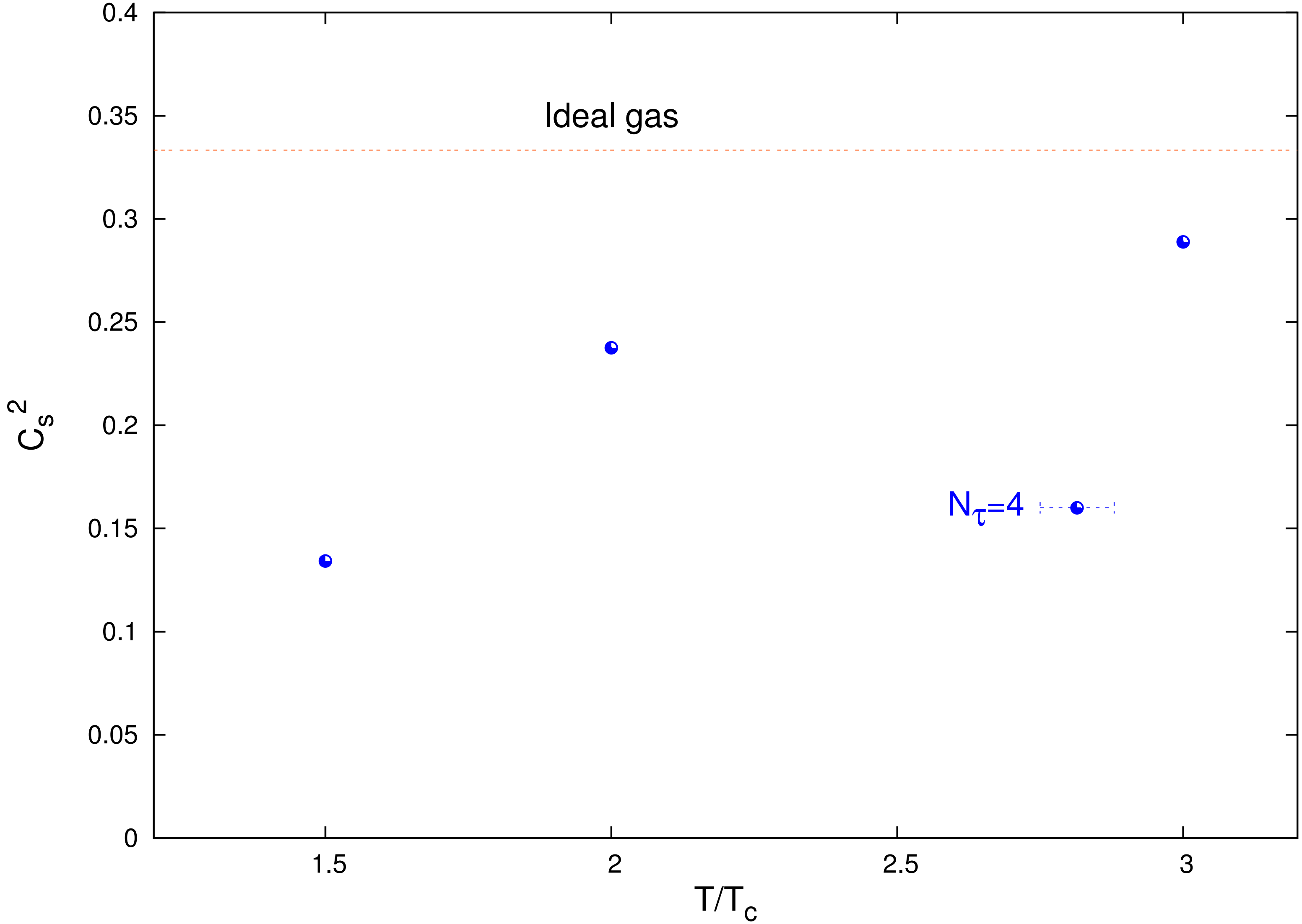
C_s^2 Results



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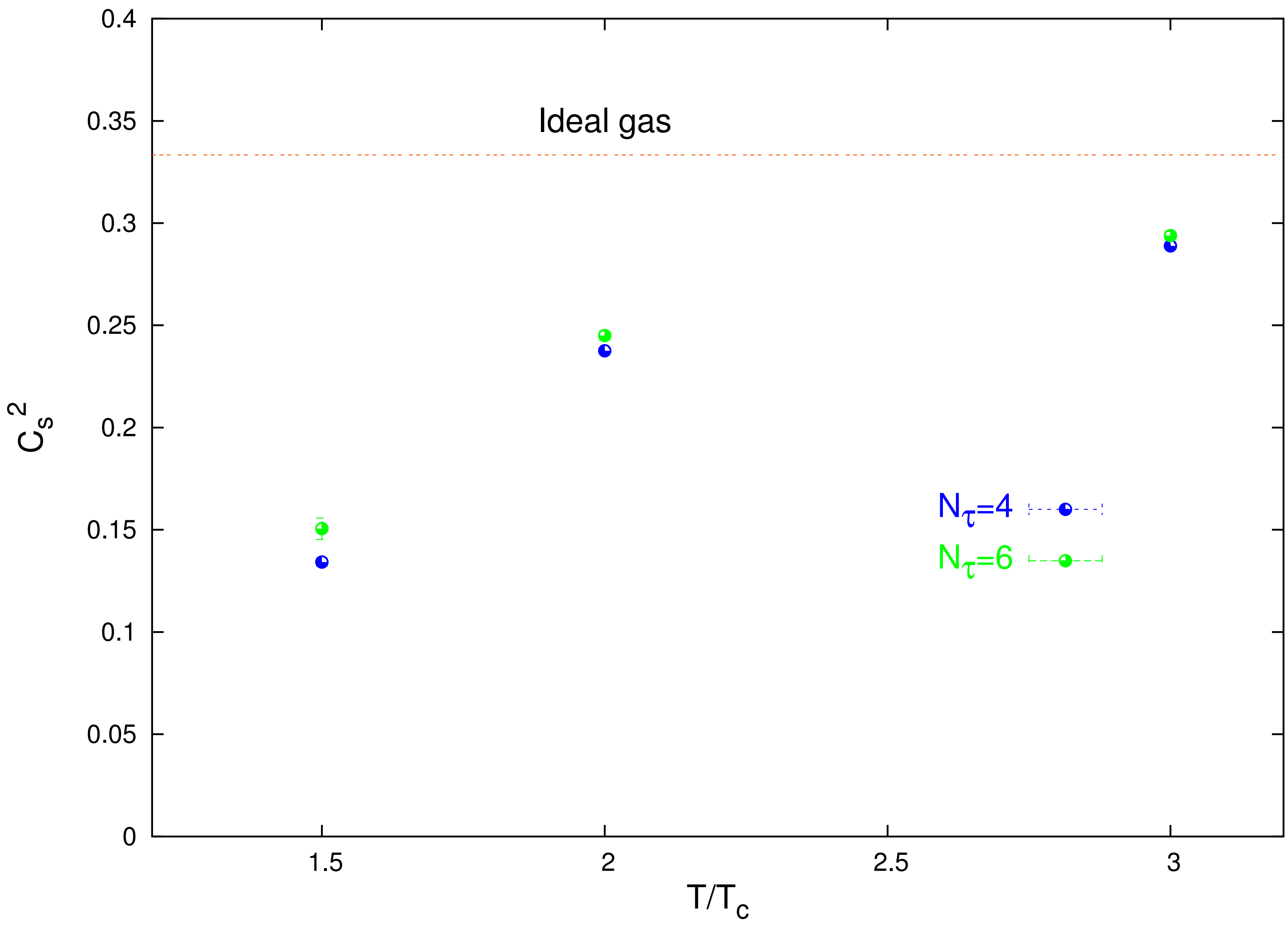
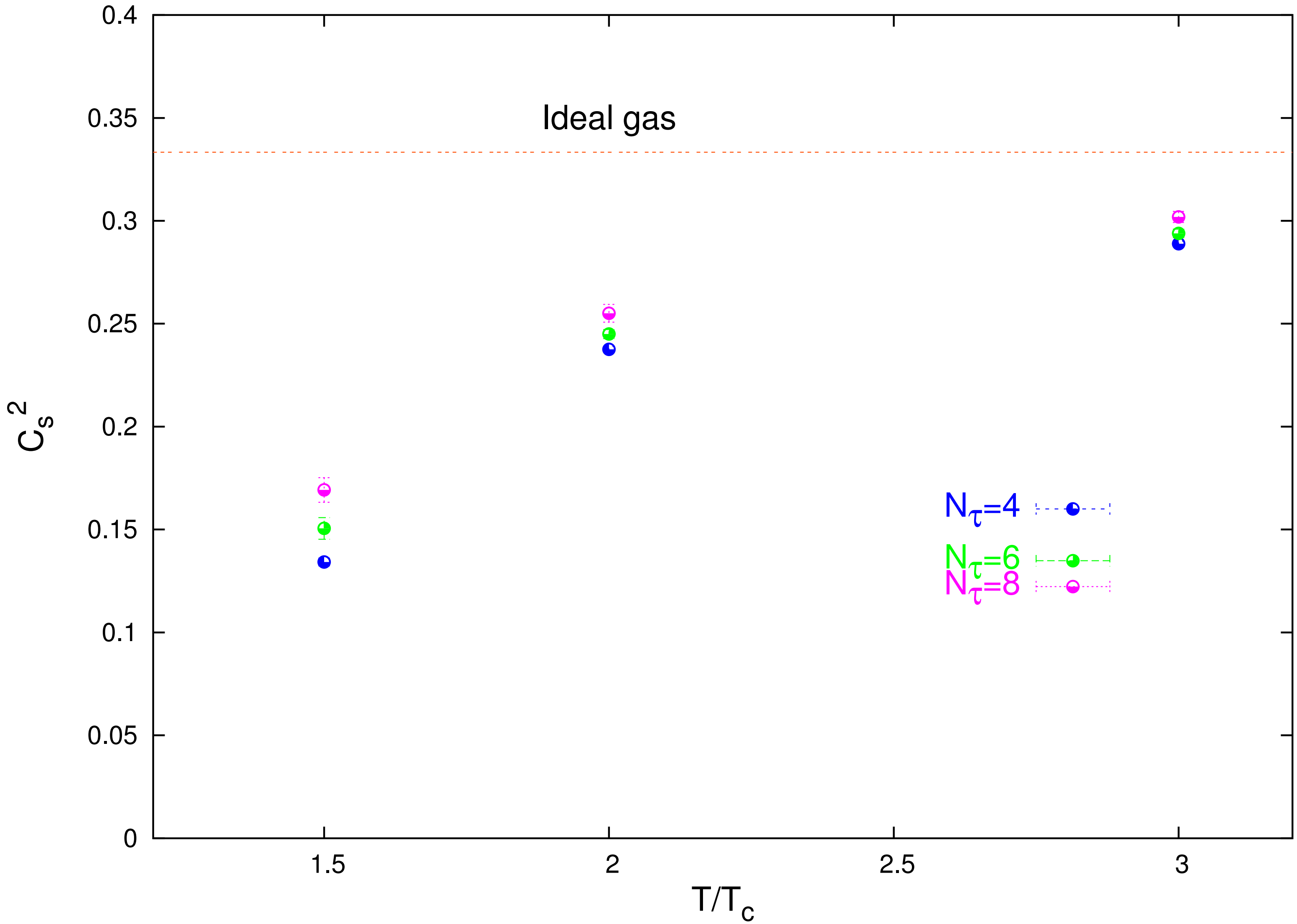

0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Ideal gas
C_s^2
N_τ=4
N_τ=6
1.5
2
2.5
3
T/T$_c$

C_s^2 Results



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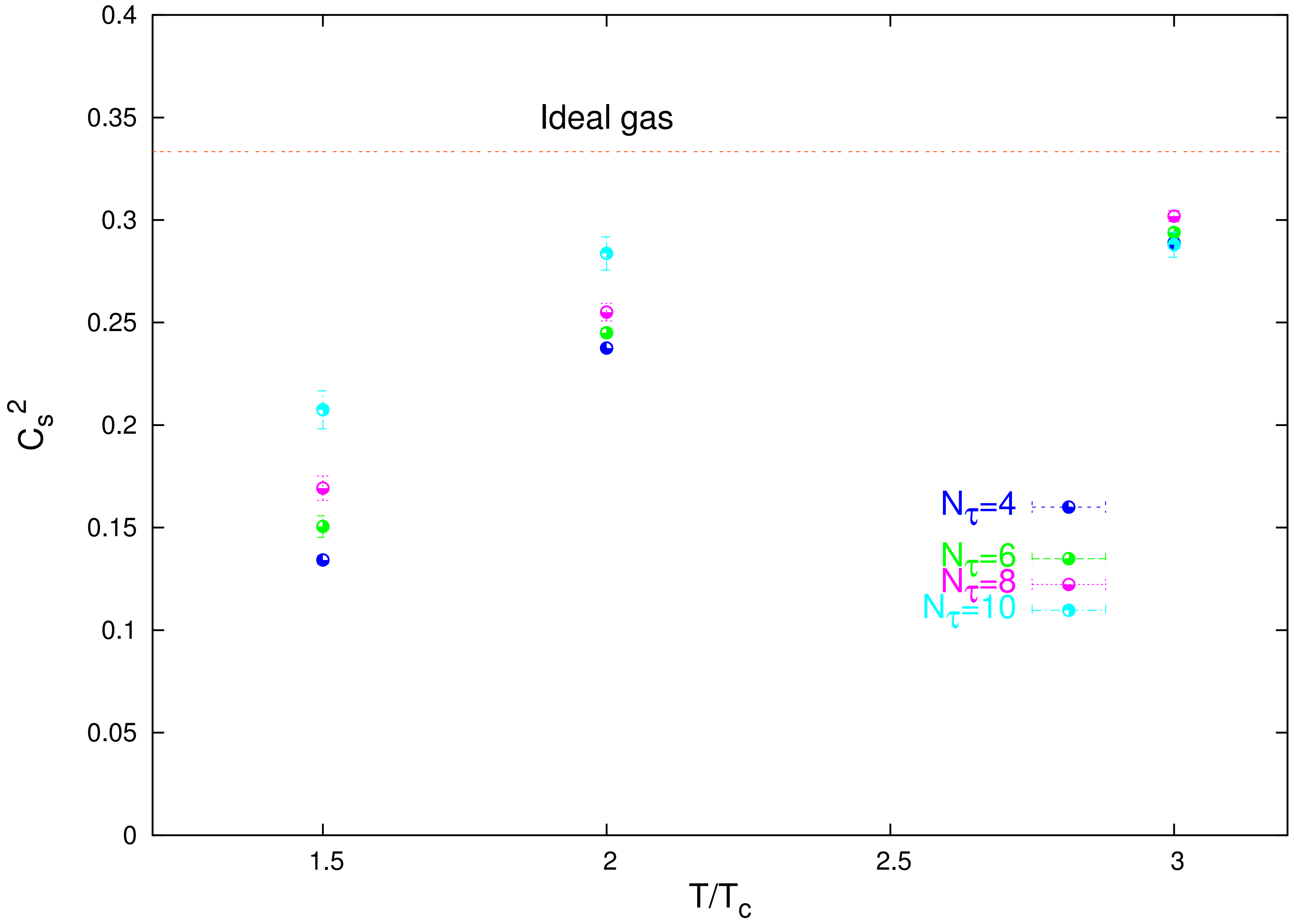
0.05
0
Ideal gas
C_s^2
N_τ=4
N_τ=6
N_τ=8
1.5
2
2.5
3
T/T_c

C_s^2 Results



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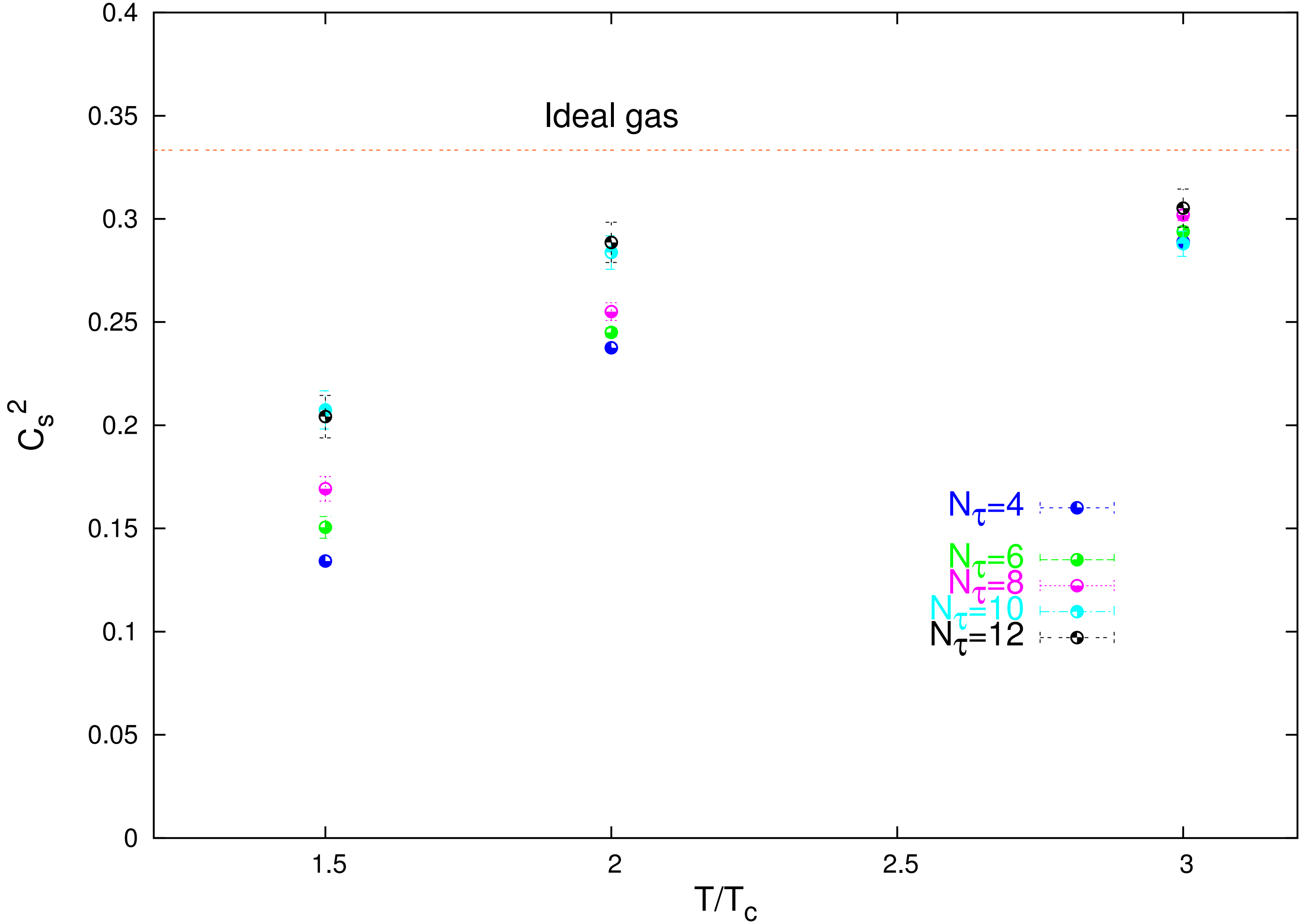
$N_\tau=8$
$N_\tau=10$

C_s^2 Results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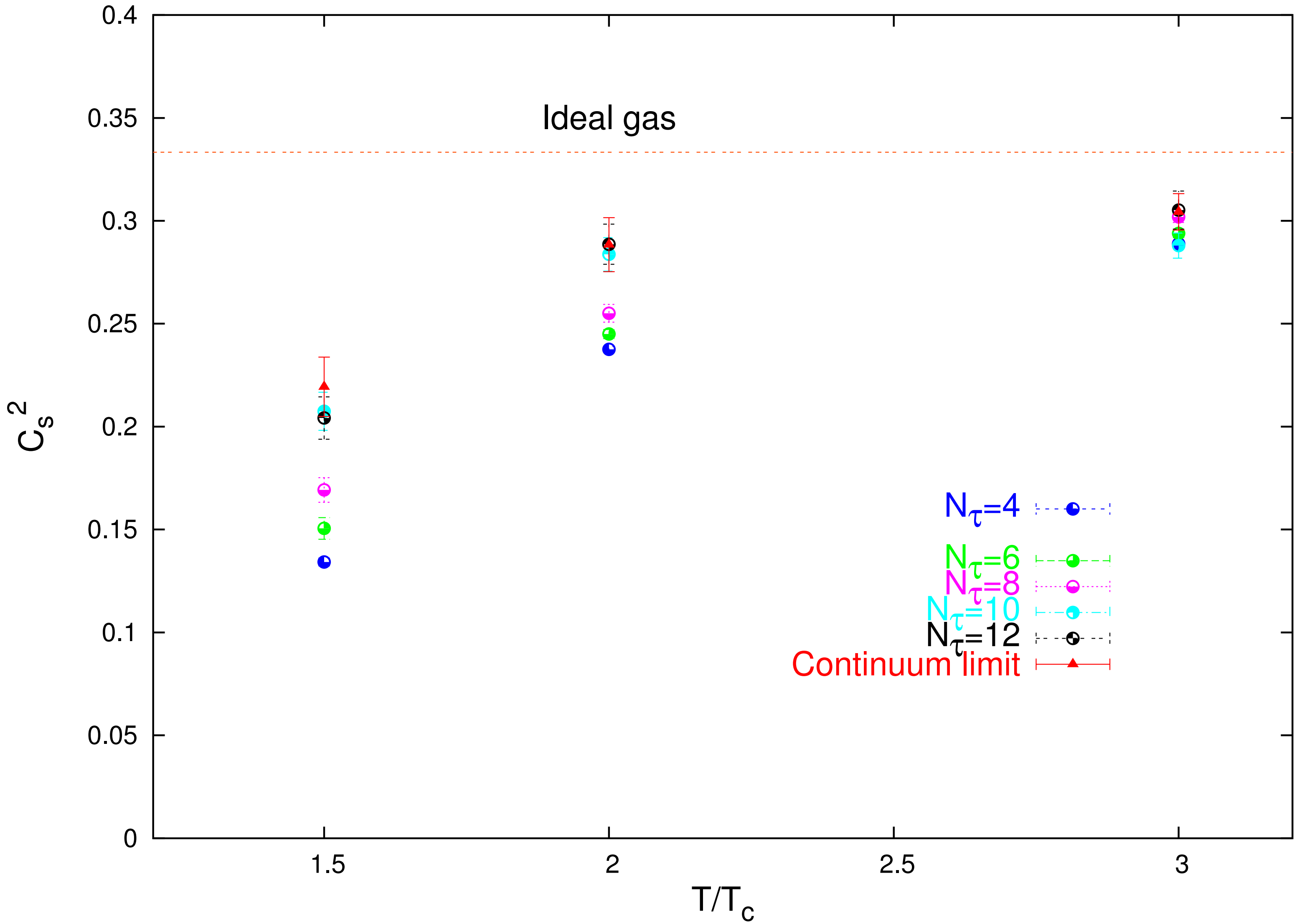
$N_\tau=8$
$N_\tau=10$
$N_\tau=12$

C_s^2 Results



Ideal gas
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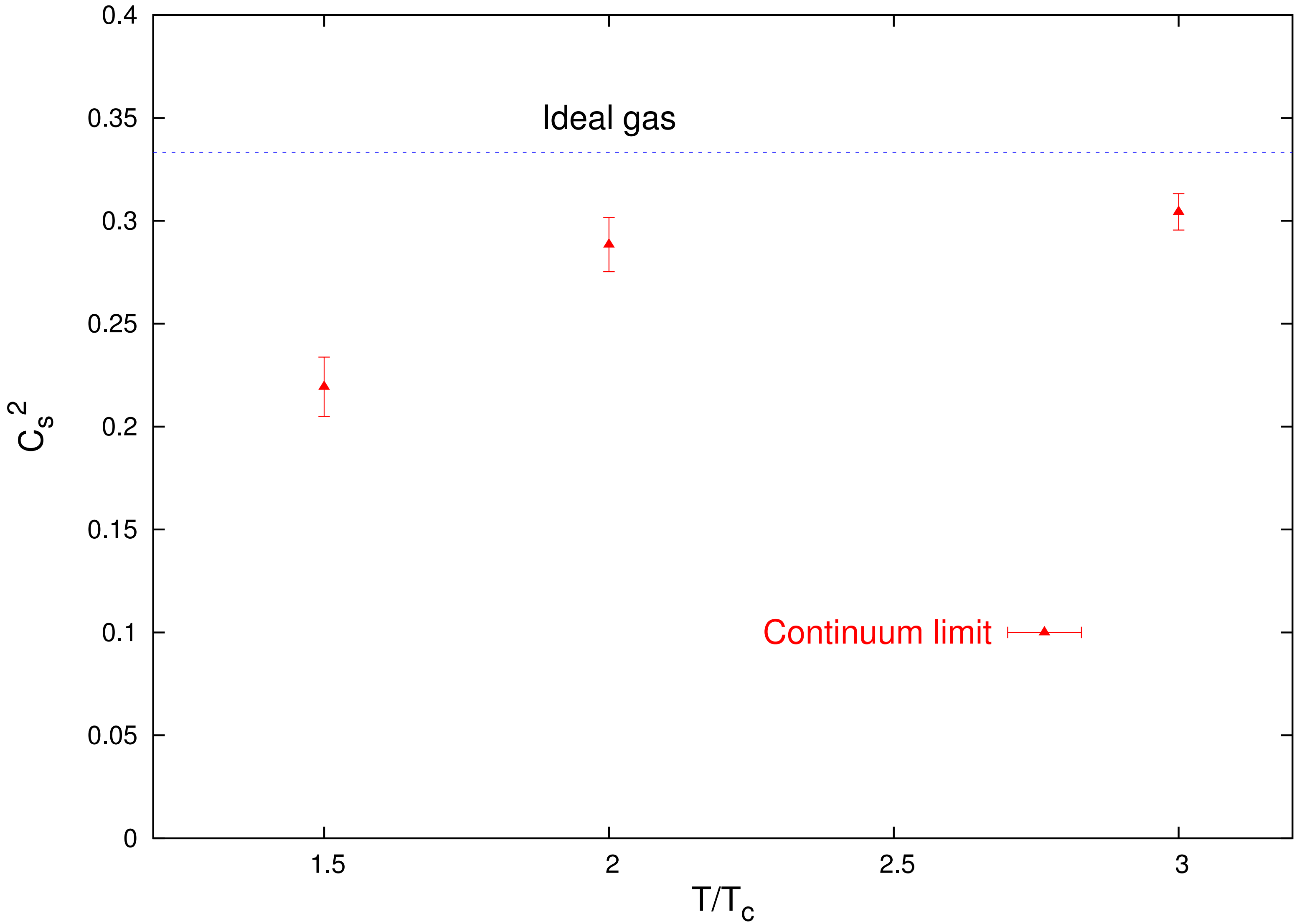
N_τ=8
N_τ=10
N_τ=12
Continuum limit

C_s^2 Results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Continuum limit
C_s^2
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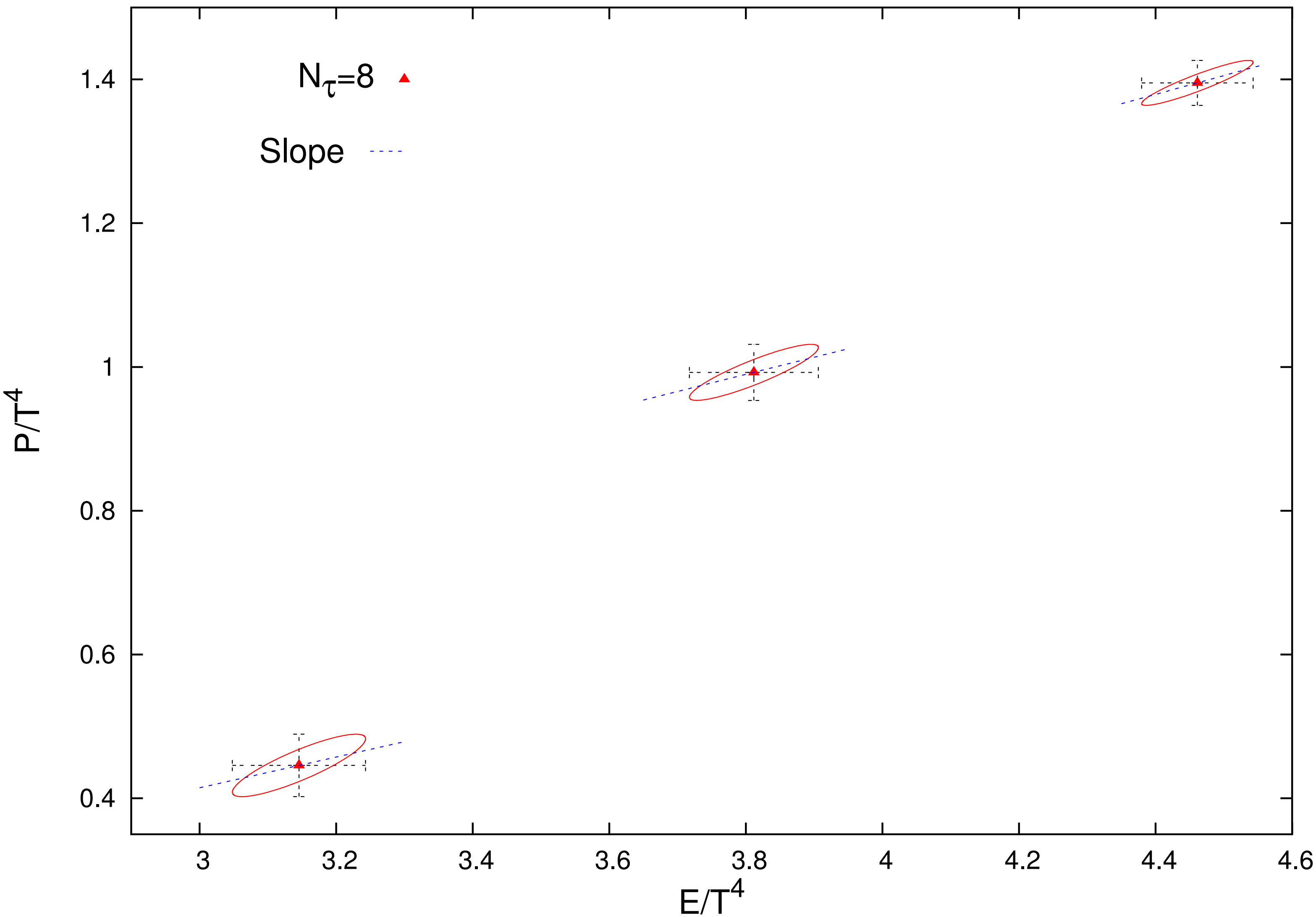
P vs E



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- N_τ are chosen such that the non-perturbative and one loop order perturbative β-functions agree.
- At $T = 3Tc$ the continuum limit of C_v differs form its value for the ideal gas by 4% with 80% confidence limit.
- At $T = 3Tc$ the continuum limit of C_s^2 differs form its value for the ideal gas by 9% with 99% confidence limit.