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No actual brains of Micros*ft engineers were harmed in the making of this presentation.


# Strings from Quivers 

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Based on:
S.M., Mukund Rangamani and Erik Verlinde, "Strings from Quivers, Membranes from Moose" hep-th/0204147
and work in progress.

Related work:
Mohsen Alishahiha and Mohammad M. Sheikh-Jabbari, "Strings in PP-waves and Worldsheet Deconstruction"
hep-th/0204174
Stephen Naculich, Howard J. Schnitzer and Niclas Wyllard, "PP-wave Limits and Orientifolds"
hep-th/0206094
Gautam Mandal, Nemani V. Suryanarayana and Spenta R. Wadia, "Aspects of Semiclassical Strings in $A d S_{5}$ "
hep-th/0206103

## Plan of the talk:

1. Outline: Quivers and DLCQ
2. Setting: Large Quiver Theories and PP Wave Limit
3. Proposal: Gauge Theory Description of DLCQ String
4. Dual: Non-Relativistic Strings and Membranes
5. Comments: Parameters and Couplings
6. Conclusions

## 1. Outline: Quivers and DLCQ

- In light-cone quantization of strings, it is often useful to compactify a null direction.

This leads to Discrete Light Cone Quantization (DLCQ) of the string theory. In this description, the theory splits into sectors labelled by a discrete value of the quantized light-cone momentum.

Interacting strings carry these quantized light-cone momenta, with the minimal momentum being carried by a "string bit".

Such a program, for the gauge theory/pp-wave correspondence, could lead to a better understanding of string interactions.

- The setting for the present talk is type IIB string theory, which admits supersymmetric solutions of the type $A d S_{5} \times M_{5}$ where $M_{5}$ is a SasakiEinstein space.
- Unfortunately, the pp-wave metric, as usually derived from $A d S_{5} \times M_{5}$, describes a noncompact null direction $x^{-}$.

- In this talk, I will show that there is a novel scaling limit of a particular AdS background, in which one ends up with a pp-wave with a compact light-cone direction.

The radius of the null direction is a finite, controllable parameter of this background.

- This particular AdS background has a dual 4d conformal gauge theory. The above scaling limit will act on this gauge theory, leading to a dual gauge theory/pp-wave pair.
- In the gauge theory, our scaling limit will play a role similar to the nowfamiliar double scaling limit in the usual BMN picture:

$$
N \rightarrow \infty, \quad J \rightarrow \infty, \quad \frac{J}{\sqrt{N}} \text { fixed }
$$

except that our limit will be taken on the theory rather than on the observables under study.

- The gauge theory in question is an $\mathcal{N}=2$ superconformal "moose" or "quiver" theory in the large moose limit.

- Several fascinating aspects of the gauge theory/pp-wave correspondence will emerge as we explore this background.

We will find gauge theory operators that can be identified with a string ground state in every sector of fixed DLCQ momentum $k$.

We will also find operators that describe modes of the string winding $m$ times on the DLCQ direction.

These operators satisfy the relation

$$
\sum_{i} n_{i}=L_{0}-\bar{L}_{0}=k m
$$

- The DLCQ theory can be T-dualized into a type IIA/M-theory background which describes a non-relativistic string/membrane bound in a harmonicoscillator potential.

Thus, the gauge theory "deconstructs" nonrelativistic IIA/M-theory in this limit.

## 2. Setting: Large Quiver Theories and PP Wave Limit

- The gauge theory that we will study is obtained by placing $N_{1} D 3$-branes transverse to the 6 -dimensional space $R^{2} \times\left(C^{2} / Z_{N_{2}}\right)$.
- The orbifold group $Z_{N_{2}}$ acts on $R^{2} \times \mathrm{C}^{2}$ by:

$$
\left(z_{1}, z_{2}, z_{3}\right) \rightarrow\left(z_{1}, \omega z_{2}, \omega^{-1} z_{3}\right), \quad \omega=e^{\frac{2 \pi i}{N_{2}}}
$$

- The theory on the brane world-volume is a $\mathcal{N}=2$ superconformal field theory in four dimensions.
- The R-symmetry group is $U(1)_{R} \times S U(2)_{R}$.
- The gauge group is $S U\left(N_{1}\right)^{(1)} \times S U\left(N_{1}\right)^{(2)} \times \cdots S U\left(N_{1}\right)^{\left(N_{2}\right)}$.
- The fields in the vector multiplet for each factor of the gauge group are denoted $\left(A_{\mu I}, \Phi_{I}, \Psi_{a I}\right)$, with $I=1,2, \ldots, N_{2}$ and $a=1,2$.
- In addition, there are hypermultiplets $\left(A_{I}, B_{I}, \chi_{a I}\right)$.
- For fixed index $I$, the $A_{I}$ and $B_{I}$ are bi-fundamentals of $S U\left(N_{1}\right)^{(I)} \times$ $\operatorname{SU}\left(N_{1}\right)^{(I+1)}$ :

$$
A_{I}:\left(1, \ldots, N_{1}, \bar{N}_{1}, \ldots, 1\right), \quad B_{I}:\left(1, \ldots, \bar{N}_{1}, N_{1}, \ldots, 1\right)
$$

- This can be represented in a "moose" or "quiver" diagram:

- The holographic dual is type IIB string theory on $A d S_{5} \times S^{5} / Z_{N_{2}}$.
- The $A d S_{5}$ space has a radius given by:

$$
R^{2}=\sqrt{4 \pi g_{s}^{B} \alpha^{\prime 2} N_{1} N_{2}}
$$

where $g_{s}^{B}$ is the type IIB string coupling.

- We are interested in a scaling limit when both $N_{1}$ and $N_{2}$ become large together, with the ratio $N_{1} / N_{2}$ fixed.
- We will see that $N_{2} \rightarrow \infty$ is like the "continuum limit" $J \rightarrow \infty$ (cf. Maldacena's talk).
- To obtain the Penrose limit, one has to focus on the trajectory of a lightlike worldline based at the origin of $A d S_{5}$.
- Because of the singular nature of the compact manifold, the result depends on the choice of this trajectory.
- We parametrize the complex coordinates of the transverse space in terms of angles:

$$
z_{1}=R \sin \alpha e^{i \theta}, \quad z_{2}=R \cos \alpha \cos \gamma e^{i \chi}, \quad z_{3}=R \cos \alpha \sin \gamma e^{i \phi}
$$

- The orbifold is obtained by demanding that $\chi$ and $\phi$ are periodic modulo $2 \pi$, and in addition have a combined periodicity under

$$
\chi \rightarrow \chi+\frac{2 \pi}{N_{2}}, \quad \phi \rightarrow \phi-\frac{2 \pi}{N_{2}} .
$$

- Now we can write the metric of $A d S_{5} \times S^{5} / \mathrm{Z}_{N_{2}}$ :

$$
\begin{aligned}
d s^{2} & =R^{2}\left[-\cosh ^{2} \rho d t^{2}+d \rho^{2}+\sinh ^{2} \rho d \Omega_{3}^{2}\right. \\
& \left.+d \alpha^{2}+\sin ^{2} \alpha d \theta^{2}+\cos ^{2} \alpha\left(d \gamma^{2}+\cos ^{2} \gamma d \chi^{2}+\sin ^{2} \gamma d \phi^{2}\right)\right]
\end{aligned}
$$

- To take the pp-wave limit, define new coordinates $r, x, y$ by:

$$
r=\rho R, \quad x=\alpha R, \quad y=\gamma R
$$

and introduce the lightcone coordinates

$$
x^{+}=\frac{1}{2}(t+\chi), \quad x^{-}=\frac{R^{2}}{2}(t-\chi)
$$

- In the limit $R \rightarrow \infty$ the metric reduces to

$$
\begin{aligned}
d s^{2} & =-4 d x^{+} d x^{-}-\left(r^{2}+x^{2}+y^{2}\right) d x^{+2}+d r^{2}+r^{2} d \Omega_{3}^{2} \\
& +d x^{2}+x^{2} d \theta^{2}+d y^{2}+y^{2} d \phi^{2} \\
& =-4 d x^{+} d x^{-}-\sum_{i=1}^{8}\left(x^{i}\right)^{2} d x^{+2}+\sum_{i=1}^{8} d x^{i^{2}}
\end{aligned}
$$

- Although we obtained the standard pp-wave metric in the Penrose limit, there is actually an important difference: the lightlike direction $x^{-}$is compact.
- To see this, note that the combined periodicity of the angles $\chi, \phi$ translates into the following periodicity on the new coordinates:

$$
x^{+} \rightarrow x^{+}+\frac{\pi}{N_{2}}, \quad x^{-} \rightarrow x^{-}+\frac{\pi R^{2}}{N_{2}}, \quad \phi \rightarrow \phi-\frac{2 \pi}{N_{2}}
$$

- Since $R^{2}=\sqrt{4 \pi g_{s}^{B} \alpha^{\prime 2} N_{1} N_{2}}$, we find that:

$$
\frac{R^{2}}{N_{2}}=2 \alpha^{\prime} \sqrt{\pi g_{s}^{B} \frac{N_{1}}{N_{2}}} \equiv 2 R_{-}
$$

Thus as $N_{1}, N_{2} \rightarrow \infty$ together, $x^{-}$is periodic with period $2 \pi R_{-}$, where:

$$
R_{-} \equiv \frac{\alpha^{\prime}}{2} g_{\mathrm{YM}} \sqrt{\frac{N_{1}}{N_{2}}}
$$

- Since the lightlike direction $x^{-}$is periodic, the corresponding light-cone momentum $p^{+}$is quantized in units of $\frac{1}{R_{-}}$.
- In other words, we are doing a Discrete Light-Cone Quantization (DLCQ) of the string on a pp-wave background.
- As is well-known, the theory then splits into sectors, labelled by the discrete number of light-cone quanta $k$. This is always a positive integer.
- There can also be winding modes of the string on the null direction, which we label by an integer $m$.


Momentum


## 3. Proposal: Gauge Theory Description of DLCQ String

- We now address the construction of string states in the DLCQ pp-wave background starting from the moose/quiver gauge theory.
- The first step is to identify the desired quantum numbers. Recall that

$$
H=2 p^{-}=\Delta-J_{\chi}
$$

where $J_{\chi}$ generates rotations of the angle $\chi$ that appears in:

$$
x^{+}=\frac{1}{2}(t+\chi), \quad x^{-}=\frac{R^{2}}{2}(t-\chi)
$$

- What is this generator in the gauge theory? It is easy to see that

$$
J_{\chi}=N_{2} J+J^{\prime}
$$

where

$$
\begin{array}{llll}
J: & A_{I} \rightarrow e^{\frac{i \beta}{2 N_{2}}} A_{I}, & B_{I} \rightarrow e^{-\frac{i \beta}{2 N_{2}}} B_{I} & \text { (global non-R symmetry) } \\
J^{\prime}: & A_{I} \rightarrow e^{\frac{i \beta}{2}} A_{I}, & B_{I} \rightarrow e^{\frac{i \beta}{2}} B_{I} & (\mathrm{R}-\text { symmetry })
\end{array}
$$

- Thus we have:

$$
\begin{aligned}
H=2 p^{-} & =\Delta-N_{2} J-J^{\prime} \\
2 p^{+} & =\frac{\Delta+N_{2} J+J^{\prime}}{R^{2}}
\end{aligned}
$$

- Note that the fundamental fields have no anomalous dimensions in this theory. So the dimensions $\Delta$ are their free-field values.
- Thus we find:

|  | $\Delta$ | $J$ | $J^{\prime}$ | $H$ |
| :---: | :---: | :---: | :---: | :---: |
| $A_{I}$ | 1 | $\frac{1}{2 N_{2}}$ | $\frac{1}{2}$ | 0 |
| $B_{I}$ | 1 | $-\frac{1}{2 N_{2}}$ | $\frac{1}{2}$ | 1 |
| $\Phi_{I}$ | 1 | 0 | 0 | 1 |
| $\bar{A}_{I}$ | 1 | $-\frac{1}{2 N_{2}}$ | $-\frac{1}{2}$ | 2 |
| $\bar{B}_{I}$ | 1 | $\frac{1}{2 N_{2}}$ | $-\frac{1}{2}$ | 1 |
| $\Phi_{I}$ | 1 | 0 | 0 | 1 |

- Now we can construct the gauge theory operator that corresponds to the ground state of the dual string theory.
- It must have $H=0$, therefore it has to be constructed out of the $A_{I}$ alone.
- As these fields are bi-fundamentals, the simplest gauge-invariant operator that can be made out of them is:

$$
\operatorname{tr}\left(A_{1} A_{2} \cdots A_{N_{2}}\right)
$$

- This operator has $H=0$ and $\Delta=N_{2}$. This implies that

$$
2 p^{+}=2 \frac{N_{2}}{R^{2}}=\frac{1}{R_{-}}
$$

- Hence we can identify it with the string ground state in the sector with one unit of DLCQ momentum (and no winding):

$$
|k=1, m=0\rangle=\frac{1}{\sqrt{\mathcal{N}}} \operatorname{tr}\left(A_{1} A_{2} \cdots A_{N_{2}}\right)
$$

- Pictorially, this operator is a "string" of fields that are "winding" around the quiver diagram.

- But in the string theory this is a momentum state!

We will see later that this has a beautiful physical interpretation.

- Now it is easy to construct all the DLCQ momentum states. We have:

$$
|k, m=0\rangle=\frac{1}{\sqrt{\mathcal{N}^{k}}} \operatorname{tr}\left(A_{1} A_{2} \cdots A_{N_{2}}\right)^{k}
$$

for any positive integer $k$. They all have $H=0$.

- For example, the state $|k=2, m\rangle$ looks like:

- The next step is to construct the zero-mode string oscillator states. These should have light-cone Hamiltonian $H=1$.
- From the table, it is clear that we can admit precisely one insertion of:

$$
\Phi_{I}, \Phi_{I}, B_{I}, B_{I}
$$

each of which has $H=1$.

- The representations of these fields:

$$
\Phi: \text { adjoint, } \quad B: \mathrm{bi}-\text { fundamental }
$$

constrain what gauge-invariant operators can be written down.

- Some of the $H=1$ operators constructed in this way are illustrated as follows:


Insertion of $\Phi$


Insertion of $\bar{\Phi}$

- For example, on the $k=1$ DLCQ ground state we can build the operator:

$$
\operatorname{tr}\left(A_{1} A_{2} \cdots A_{I-1} \Phi_{I} A_{I} \cdots A_{N_{2}}\right)
$$

- Invariance under the orbifold group $Z_{N_{2}}$ is achieved by summing over the insertion point. Thus we propose:

$$
a_{\Phi, 0}^{\dagger}|k=1, m=0\rangle \sim \sum_{I=1}^{N_{2}} \operatorname{tr}\left(A_{1} A_{2} \cdots A_{I-1} \Phi_{I} A_{I} \cdots A_{N_{2}}\right)
$$

and similarly for $\bar{\Phi}, A_{I} B_{I}, \bar{B}_{I}$.

- Another four are given by:

$$
\partial_{i} \operatorname{tr}\left(A_{1} A_{2} \cdots A_{N_{2}}\right)
$$

Thus for $k=1$, we have identified the 8 zero-mode bosonic oscillators of the string. For general $k$, the construction is analogous.

- String oscillators with nonzero mode number can be obtained by inserting phases.
- However, we will see that in our model, this automatically introduces winding states as well.
- In DLCQ string theories, it is well-known that the constraint $L_{0}-\bar{L}_{0}=0$ is replaced, in the sector of momentum $k$ and winding $m$, by:

$$
L_{0}-\bar{L}_{0}=k m
$$

- This leads to the identification:

$$
a_{\Phi, m}^{\dagger}|k=1, m\rangle=\sum_{I=1}^{N_{2}} \operatorname{tr}\left(A_{1} A_{2} \cdots A_{I-1} \Phi_{I} A_{I} \cdots A_{N_{2}}\right) \omega^{m I}
$$

where

$$
\omega=e^{\frac{2 \pi i}{N_{2}}}
$$

- The general winding state in the sector of DLCQ momentum 1 is then:

$$
\prod_{i=1}^{M} a_{\Phi, n_{i}}^{\dagger}|k=1, m\rangle=\sum_{l_{M} \geq \cdots \geq l_{2} \geq l_{1}}^{N_{2}} \operatorname{tr}\left(A_{1} \cdots A_{l_{1}-1} \Phi_{l_{1}} \cdots A_{l_{i}-1} \Phi_{l_{i}} A_{l_{i}} \cdots A_{N_{2}}\right) \omega^{\sum n_{i} l_{i}}
$$

where the winding number $m$ is defined as the sum of the mode numbers $n_{i}$ :

$$
m \equiv \sum_{i} n_{i}
$$

- The gauge theory operators that describe the sector with DLCQ momentum $k>1$ are generalizations of the above. For example,

$$
a_{\Phi, n}^{\dagger}|k=2, m\rangle=\sum_{I=1}^{2 N_{2}} \operatorname{tr}\left(A_{1} \cdots A_{I-1} \Phi_{I} A_{I} \cdots A_{N_{2}} A_{1} A_{2} \cdots A_{N_{2}}\right) \omega^{n I}
$$

where now

$$
\omega=e^{\frac{2 \pi i}{2 N_{2}}} \quad \text { and } \quad m=\frac{n}{2}
$$

It is easy to show that the above state vanishes for odd $n$, an important consistency check.

- We see that the DLCQ winding states, represented as gauge theory operators, look like momentum states on the large moose.
- And we already saw that the DLCQ momentum states look like winding states on the large moose.
- This is suggestive of T-duality.


## 4. Dual: Non-Relativistic Strings and Membranes

- One can gain more insight into our construction by performing a T-duality over the lightlike DLCQ direction.
- The periodicity of the $x^{-}$direction is a remnant of the combined periodicities in the angles $\chi$ and $\phi$, exhibited earlier.
- Before taking the limit $N_{1}, N_{2} \rightarrow \infty$, the periodic direction was space-like. Hence one can perform a T-duality along this direction.
- Let us go back to the original $A d S_{5} \times S^{5} / Z_{N_{2}}$ metric and write down only the terms in the $t$ and $\chi$ directions (i.e., ignoring the transverse space):

$$
d s^{2}=R^{2}\left[-\cosh ^{2} \rho d t^{2}+\cos ^{2} \alpha \cos ^{2} \gamma d \chi^{2}\right]
$$

- Now we make the original replacements for $\chi, \rho, \alpha, \gamma$ in terms of the pp-wave-adapted coordinates:

$$
x^{-}=\frac{R^{2}}{2}(t-\chi), \quad r=\rho R, \quad x=\alpha R, \quad y=\gamma R
$$

- We do not yet take the limit $R \rightarrow \infty$, so the metric becomes:

$$
\begin{aligned}
d s^{2} & =R^{2}\left(\cos ^{2} \frac{w}{R} \cos ^{2} \frac{y}{R}-\cosh ^{2} \frac{r}{R}\right) d t^{2}-4 \cos ^{2} \frac{w}{R} \cos ^{2} \frac{y}{R} d t d x^{-} \\
& +\frac{4}{R^{2}} \cos ^{2} \frac{w}{R} \cos ^{2} \frac{y}{R}\left(d x^{-}\right)^{2}
\end{aligned}
$$

- This procedure has introduced a small $g_{--}$in the metric, and we can now T-dualize. We end up with the metric:

$$
d s^{2}=-R^{2} \cosh ^{2} \frac{r}{R} d t^{2}+\frac{R^{2}}{\cos ^{2} \frac{w}{R} \cos ^{2} \frac{y}{R}}\left(d x^{9}\right)^{2}
$$

along with a B-field and dilaton:

$$
B_{t 9}=-R^{2}, \quad g_{s}^{A}=\frac{\sqrt{\alpha^{\prime}} R}{R_{-} \cos \frac{w}{R} \cos \frac{y}{R}}
$$

Here $2 x^{9}$ is the T-dual of $x^{-}$.

- Note that $x^{9}$ now has period $2 \pi \frac{\alpha^{\prime}}{R_{-}}$, with as before:

$$
R_{-}=\frac{\alpha^{\prime}}{2} g_{\mathrm{YM}} \sqrt{\frac{N_{1}}{N_{2}}}
$$

- There are also Ramond-Ramond fields that we do not write here.
- Evidently some components of the metric, and the $B$-field and string coupling, become infinite as $R \rightarrow \infty$.
- However, string propagation on this background is finite. The reason is that the $B$-field is a critical electric field and cancels the leading divergent piece in the string world-sheet action:

$$
\sqrt{-\operatorname{det}(g)}+B=R^{2} \frac{\cosh \frac{r}{R}}{\cos \frac{w}{R} \cos \frac{y}{R}}-R^{2} \simeq \frac{1}{2} \sum_{i=1}^{8}\left(x^{i}\right)^{2}+\mathcal{O}\left(\frac{1}{R^{2}}\right)
$$

- This is just the non-relativistic string propagating in a background with a Newtonian potential of harmonic-oscillator type.
- It is well-known that in a critical electric field with the above scaling, closed strings winding in the direction of the field are light, while the others are are heavy.
- So the non-relativistic closed string (NRCS) that we have arrived at, has only positive windings over the circle.
- The interpretation of our gauge-theory operators winding round the moose is now clear. They are deconstructing these winding states of the NRCS:


Operators winding around moose


NR string winding around spatial circle

- The string of $A$ 's had vanishing energy for any winding number, like the light winding strings of NRCS theory.
- The string of $\bar{A}$ 's winding the other way gives infinitely energetic states as $N_{2} \rightarrow \infty$, like the NRCS strings winding the wrong way.
- Insertions of phases give the (quantized) momentum states of the NR closed string on $x^{9}$. These can have either sign of momentum.

5. Comments: Parameters and Couplings
(i) Effective 't Hooft coupling:

In the $\mathcal{N}=4$ case, this is:

$$
\lambda^{\prime}=\frac{g_{\mathrm{YM}}^{2} N}{J^{2}}
$$

In the quiver theory we expect:

$$
N \rightarrow N_{1} N_{2}, \quad J \rightarrow k N_{2}
$$

hence

$$
\lambda^{\prime}=\frac{g_{\mathrm{YM}}^{2}}{k^{2}} \frac{N_{1}}{N_{2}}=\left(\frac{2 R_{-}}{k}\right)^{2}
$$

where we recall that

$$
\left.R_{-} \equiv \frac{1}{2} g_{\mathrm{YM}} \sqrt{\frac{N_{1}}{N_{2}}} \quad \text { (in } \alpha^{\prime}=1 \text { units }\right)
$$

(ii) Genus expansion parameter:

In the $\mathcal{N}=4$ theory, this is:

$$
g_{2}=\frac{J^{2}}{N}
$$

In the quiver theory, the corresponding object should be:

$$
g_{2}=k^{2} \frac{N_{2}}{N_{1}} \sim \frac{N_{2}}{N_{1}}
$$

This can be checked directly by computing all-genus correlators in the free quiver gauge theory. For example, one finds:
$\langle 0| \operatorname{tr}\left(A_{1} A_{2} \ldots A_{N_{2}}\right)^{k} \operatorname{tr}\left(\bar{A}_{N_{2}} \bar{A}_{N_{2}-1} \ldots \bar{A}_{1}\right)^{k^{\prime}}|0\rangle=\frac{\delta_{k, k^{\prime}}}{|x|^{2 N_{2}}} \sum_{l=1}^{k}\left(\frac{\Gamma\left(N_{1}+l\right)}{\Gamma\left(N_{1}+l-k\right)}\right)^{N_{2}}$

As $N_{1}, N_{2} \rightarrow \infty$ with fixed ratio, this reduces to:

$$
\frac{\delta_{k, k^{\prime}}}{|x|^{2 N_{2}}} \times 2 \sum_{l=1}^{\frac{k}{2}} \cosh \left\{\left(l-\frac{(k+1)}{2}\right) k \frac{N_{2}}{N_{1}}\right\}
$$

This has an expansion in powers of

$$
\left(\frac{N_{2}}{N_{1}}\right)^{2}
$$

as expected.

## (iii) Effective coupling:

In the $\mathcal{N}=4$ theory there is a combination that describes the effective coupling between states of the same $\Delta-J$ (at small $\lambda^{\prime}$ ):

$$
g_{\mathrm{eff}}=g_{2} \sqrt{\lambda^{\prime}}=g_{\mathrm{YM}} \frac{J}{\sqrt{N}}
$$

The corresponding effective coupling in the quiver case would then be:

$$
g_{\mathrm{eff}}=g_{\mathrm{YM}} k \sqrt{\frac{N_{2}}{N_{1}}}
$$

Let us compare this with the type IIA NR closed string coupling, given by the familiar (NCOS) formula:

$$
g_{\mathrm{NR}}=g_{\mathrm{NCOS}}^{2}=g_{s}^{A} \sqrt{\frac{\operatorname{det}(g+B)}{\operatorname{det} g}}
$$

If we evaluate this on our background, we find a spatially varying coupling:

$$
g_{\mathrm{NR}}=\frac{g_{s}^{B}}{R_{-}} \sqrt{\sum_{i=1}^{8}\left(x^{i}\right)^{2}}
$$

- Since the string is trapped in a harmonic potential, the states are confined to a finite region, and the coupling constant will reduce to:

$$
g_{\mathrm{NR}} \sim \frac{g_{s}^{B}}{R_{-}} \sim g_{\mathrm{YM}} \sqrt{\frac{N_{2}}{N_{1}}}=g_{\mathrm{eff}}
$$

We see that the effective coupling among a subclass of gauge theory states can be identified with the effective coupling between the winding NR closed strings.

- By taking this coupling to be large, we go over to M-theory. The wound NR string becomes a non-relativistic membrane wound over the $x^{9}, x^{10}$ directions.
(iv) pp-wave mass parameter $\mu$ :

In the noncompact (usual) pp-wave background, the parameter $\mu$ can be scaled to any value by rescaling

$$
x^{+} \rightarrow \wedge x^{+}, \quad x^{-} \rightarrow \Lambda^{-1} x^{-}
$$

for some $\Lambda$.
It is also true that in a flat-space DLCQ background, the DLCQ radius can be scaled to any value (by the same procedure).

But in the DLCQ pp-wave, things are different. We can simultaneously change $\mu$ and the DLCQ radius $R_{-}$by the above scaling, but not either one separately. This is why there is one physically relevant parameter.

## (v) Deconstruction:

The same quiver gauge theory that we have been discussing was studied last year in a different limit: $N_{2} \rightarrow \infty, N_{1}$ fixed, and $\alpha^{\prime} \rightarrow 0$.

It was proposed that in this limit, taken along the Higgs branch, two additional dimensions are dynamically deconstructed and one ends up with the $(2,0)$ field theory. This arises as the decoupled theory on M5-branes.

We take $N_{1}, N_{2} \rightarrow \infty$ together with the near-horizon limit of the D3-branes. Also, we zoom in on a lightlike geodesic. This is similar to being in the Higgs branch, because we miss the orbifold singularity.

Our final result is a DLCQ pp-wave of radius $\sim \sqrt{N_{1} / N_{2}}$, or a Galilean string/membrane wrapped on a circle/torus of radius $\sim \sqrt{N_{2} / N_{1}}$.

It is not clear if conventional deconstruction can be obtained from this as $N_{2} / N_{1} \rightarrow \infty$.

## 6. Conclusions

- Large quiver theories give a nontrivial generalization of the gauge/pp-wave correspondence. They describe the Discrete Light Cone Quantization of type IIB string theory on a pp-wave background.
- They also deconstruct the non-relativistic closed string/membrane. The operator wrapping the moose once is a DLCQ string bit.
- It is clearly of interest to study string interactions in this model. For small values of $k$, the formulae should be simpler and hopefully we can check more.
- Another open problem is to directly study the non-relativistic string in a potential and compare it with gauge theory.
- But there is a more important conceptual question: DLCQ is often associated to a fundamental formulation of a theory (as with M(atrix) Theory). Is there such a fundamental formulation - M(oose) Theory hidden here?

THE END

