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# Strings from Quivers

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Based on:

S.M., Mukund Rangamani and Erik Verlinde, "Strings from Quivers, Membranes from Moose" hep-th/0204147

and work in progress.

Related work:

Mohsen Alishahiha and Mohammad M. Sheikh-Jabbari, "Strings in PP-waves and Worldsheet Deconstruction" hep-th/0204174

Stephen Naculich, Howard J. Schnitzer and Niclas Wyllard, "PP-wave Limits and Orientifolds"

hep-th/0206094

Gautam Mandal, Nemani V. Suryanarayana and Spenta R. Wadia, "Aspects of Semiclassical Strings in  $AdS_5$ " hep-th/0206103

# Plan of the talk:

- 1. Outline: Quivers and DLCQ
- 2. Setting: Large Quiver Theories and PP Wave Limit
- 3. Proposal: Gauge Theory Description of DLCQ String
- 4. **Dual: Non-Relativistic Strings and Membranes**
- 5. Comments: Parameters and Couplings
- 6. Conclusions

# 1. Outline: Quivers and DLCQ

• In light-cone quantization of strings, it is often useful to compactify a null direction.

This leads to Discrete Light Cone Quantization (DLCQ) of the string theory.

In this description, the theory splits into sectors labelled by a discrete value of the quantized light-cone momentum.

Interacting strings carry these quantized light-cone momenta, with the minimal momentum being carried by a "string bit".

Such a program, for the gauge theory/pp-wave correspondence, could lead to a better understanding of string interactions .

[5]

- The setting for the present talk is type IIB string theory, which admits supersymmetric solutions of the type  $AdS_5 \times M_5$  where  $M_5$  is a Sasaki-Einstein space.
- Unfortunately, the pp-wave metric, as usually derived from  $AdS_5 \times M_5$ , describes a noncompact null direction  $x^-$ .



[6]

• In this talk, I will show that there is a novel scaling limit of a particular AdS background, in which one ends up with a pp-wave with a compact light-cone direction.

The radius of the null direction is a finite, controllable parameter of this background.

- This particular AdS background has a dual 4d conformal gauge theory. The above scaling limit will act on this gauge theory, leading to a dual gauge theory/pp-wave pair.
- In the gauge theory, our scaling limit will play a role similar to the now-familiar double scaling limit in the usual BMN picture:

$$N o \infty, \quad J o \infty, \qquad rac{J}{\sqrt{N}} ext{ fixed}$$

except that our limit will be taken on the theory rather than on the observables under study.

• The gauge theory in question is an  $\mathcal{N} = 2$  superconformal "moose" or "quiver" theory in the large moose limit.



• Several fascinating aspects of the gauge theory/pp-wave correspondence will emerge as we explore this background.

We will find gauge theory operators that can be identified with a string ground state in every sector of fixed DLCQ momentum k.

We will also find operators that describe modes of the string winding m times on the DLCQ direction.

These operators satisfy the relation

$$\sum_{i} n_i = L_0 - \overline{L}_0 = km$$

• The DLCQ theory can be T-dualized into a type IIA/M-theory background which describes a non-relativistic string/membrane bound in a harmonic-oscillator potential.

Thus, the gauge theory "deconstructs" nonrelativistic IIA/M-theory in this limit.

#### 2. Setting: Large Quiver Theories and PP Wave Limit

- The gauge theory that we will study is obtained by placing  $N_1$  D3-branes transverse to the 6-dimensional space  $\mathbb{R}^2 \times (\mathbb{C}^2/\mathbb{Z}_{N_2})$ .
- The orbifold group  $Z_{N_2}$  acts on  $R^2 \times C^2$  by:

$$(z_1, z_2, z_3) \to (z_1, \omega z_2, \omega^{-1} z_3), \qquad \omega = e^{\frac{2\pi i}{N_2}}$$

- The theory on the brane world-volume is a  $\mathcal{N} = 2$  superconformal field theory in four dimensions.
- The R-symmetry group is  $U(1)_R \times SU(2)_R$ .
- The gauge group is  $SU(N_1)^{(1)} \times SU(N_1)^{(2)} \times \cdots SU(N_1)^{(N_2)}$ .
- The fields in the vector multiplet for each factor of the gauge group are denoted  $(A_{\mu I}, \Phi_I, \Psi_{aI})$ , with  $I = 1, 2, ..., N_2$  and a = 1, 2.

- In addition, there are hypermultiplets  $(A_I, B_I, \chi_{aI})$ .
- For fixed index I, the  $A_I$  and  $B_I$  are bi-fundamentals of  $SU(N_1)^{(I)} \times SU(N_1)^{(I+1)}$ :

 $A_{I}: (1, ..., N_{1}, \overline{N}_{1}, ..., 1), \qquad B_{I}: (1, ..., \overline{N}_{1}, N_{1}, ..., 1)$ 

• This can be represented in a "moose" or "quiver" diagram:  $\Phi_{I-3} \qquad \Phi_{I-1} \qquad A_{I-1} \qquad B_{I-1} \qquad B_{I$ 

- [11]
- The holographic dual is type IIB string theory on  $AdS_5 \times S^5/Z_{N_2}$ .
- The  $AdS_5$  space has a radius given by:

$$R^2 = \sqrt{4\pi g_s^B \alpha'^2 N_1 N_2}$$

where  $g_s^B$  is the type IIB string coupling.

- We are interested in a scaling limit when both  $N_1$  and  $N_2$  become large together, with the ratio  $N_1/N_2$  fixed.
- We will see that  $N_2 \to \infty$  is like the "continuum limit"  $J \to \infty$  (cf. Maldacena's talk).

[12]

- To obtain the Penrose limit, one has to focus on the trajectory of a lightlike worldline based at the origin of  $AdS_5$ .
- Because of the singular nature of the compact manifold, the result depends on the choice of this trajectory.
- We parametrize the complex coordinates of the transverse space in terms of angles:

$$z_1 = R \sin \alpha e^{i\theta}, \quad z_2 = R \cos \alpha \cos \gamma e^{i\chi}, \quad z_3 = R \cos \alpha \sin \gamma e^{i\phi}$$

• The orbifold is obtained by demanding that  $\chi$  and  $\phi$  are periodic modulo  $2\pi$ , and in addition have a combined periodicity under

$$\chi \to \chi + \frac{2\pi}{N_2}, \qquad \phi \to \phi - \frac{2\pi}{N_2} \;.$$

• Now we can write the metric of  $AdS_5 \times S^5/Z_{N_2}$ :

$$ds^{2} = R^{2} \Big[ -\cosh^{2}\rho dt^{2} + d\rho^{2} + \sinh^{2}\rho d\Omega_{3}^{2} \\ + d\alpha^{2} + \sin^{2}\alpha d\theta^{2} + \cos^{2}\alpha \left( d\gamma^{2} + \cos^{2}\gamma d\chi^{2} + \sin^{2}\gamma d\phi^{2} \right) \Big]$$

• To take the pp-wave limit, define new coordinates r, x, y by:

 $r = \rho R, \qquad x = \alpha R, \qquad y = \gamma R$ 

and introduce the lightcone coordinates

$$x^{+} = \frac{1}{2}(t + \chi), \qquad x^{-} = \frac{R^{2}}{2}(t - \chi)$$

• In the limit  $R \to \infty$  the metric reduces to

$$ds^{2} = -4dx^{+}dx^{-} - (r^{2} + x^{2} + y^{2}) dx^{+2} + dr^{2} + r^{2}d\Omega_{3}^{2}$$
  
+  $dx^{2} + x^{2}d\theta^{2} + dy^{2} + y^{2}d\phi^{2}$   
=  $-4dx^{+}dx^{-} - \sum_{i=1}^{8} (x^{i})^{2} dx^{+2} + \sum_{i=1}^{8} dx^{i^{2}}$ 

- Although we obtained the standard pp-wave metric in the Penrose limit, there is actually an important difference: the lightlike direction  $x^-$  is compact.
- To see this, note that the combined periodicity of the angles  $\chi, \phi$  translates into the following periodicity on the new coordinates:

$$x^+ \to x^+ + \frac{\pi}{N_2}, \qquad x^- \to x^- + \frac{\pi R^2}{N_2}, \qquad \phi \to \phi - \frac{2\pi}{N_2}$$

• Since  $R^2 = \sqrt{4\pi g_s^B \alpha'^2 N_1 N_2}$ , we find that:

$$\frac{R^2}{N_2} = 2\alpha' \sqrt{\pi g_s^B \frac{N_1}{N_2}} \equiv 2R_-$$

Thus as  $N_1, N_2 \rightarrow \infty$  together,  $x^-$  is periodic with period  $2\pi R_-$ , where:

$$R_{-} \equiv \frac{\alpha'}{2} g_{\rm YM} \sqrt{\frac{N_1}{N_2}}$$

- Since the lightlike direction  $x^-$  is periodic, the corresponding light-cone momentum  $p^+$  is quantized in units of  $\frac{1}{R_-}$ .
- In other words, we are doing a Discrete Light-Cone Quantization (DLCQ) of the string on a pp-wave background.
- As is well-known, the theory then splits into sectors, labelled by the discrete number of light-cone quanta k. This is always a positive integer.
- There can also be winding modes of the string on the null direction, which we label by an integer *m*.



### 3. Proposal: Gauge Theory Description of DLCQ String

- We now address the construction of string states in the DLCQ pp-wave background starting from the moose/quiver gauge theory.
- The first step is to identify the desired quantum numbers. Recall that

$$H = 2p^- = \Delta - J_{\chi}$$

where  $J_{\chi}$  generates rotations of the angle  $\chi$  that appears in:

$$x^{+} = \frac{1}{2}(t + \chi), \qquad x^{-} = \frac{R^{2}}{2}(t - \chi)$$

• What is this generator in the gauge theory? It is easy to see that  $J_{\chi} = N_2 J + J'$ 

where

$$J: A_I \to e^{\frac{i\beta}{2N_2}} A_I, \quad B_I \to e^{-\frac{i\beta}{2N_2}} B_I \qquad \text{(global non-R symmetry)}$$
$$J': A_I \to e^{\frac{i\beta}{2}} A_I, \qquad B_I \to e^{\frac{i\beta}{2}} B_I \qquad \text{(R-symmetry)}$$

• Thus we have:

$$H = 2p^{-} = \Delta - N_2 J - J'$$
$$2p^{+} = \frac{\Delta + N_2 J + J'}{R^2}$$

- Note that the fundamental fields have no anomalous dimensions in this theory. So the dimensions  $\Delta$  are their free-field values.
- Thus we find:

	Δ	J	J'	H
$A_I$	1	$\frac{1}{2N_2}$	$\frac{1}{2}$	0
$B_I$	1	$-\frac{1}{2N_2}$	$\frac{1}{2}$	1
$\Phi_I$	1	0	0	1
$\overline{A}_I$	1	$-\frac{1}{2N_2}$	$-\frac{1}{2}$	2
$\overline{B}_I$	1	$\frac{1}{2N_2}$	$-\frac{1}{2}$	1
$\overline{\Phi}_I$	1	0	0	1

- Now we can construct the gauge theory operator that corresponds to the ground state of the dual string theory.
- It must have H = 0, therefore it has to be constructed out of the  $A_I$  alone.
- As these fields are bi-fundamentals, the simplest gauge-invariant operator that can be made out of them is:

 $\mathsf{tr}\left(A_1A_2\cdots A_{N_2}\right)$ 

• This operator has H = 0 and  $\Delta = N_2$ . This implies that

$$2p^+ = 2\frac{N_2}{R^2} = \frac{1}{R_-}$$

 Hence we can identify it with the string ground state in the sector with one unit of DLCQ momentum (and no winding):

$$|k=1,m=0
angle=rac{1}{\sqrt{\mathcal{N}}} \operatorname{tr}\left(A_{1}A_{2}\cdots A_{N_{2}}
ight)$$

• Pictorially, this operator is a "string" of fields that are "winding" around the quiver diagram.



• But in the string theory this is a momentum state!

We will see later that this has a beautiful physical interpretation.

• Now it is easy to construct all the DLCQ momentum states. We have:

$$|k,m=0
angle=rac{1}{\sqrt{\mathcal{N}^k}} \mathrm{tr} \left(A_1 A_2 \cdots A_{N_2}\right)^k$$

for any positive integer k. They all have H = 0.

• For example, the state  $|k = 2, m\rangle$  looks like:



• The next step is to construct the zero-mode string oscillator states. These should have light-cone Hamiltonian H = 1.

[21]

• From the table, it is clear that we can admit precisely one insertion of:

 $\Phi_{I}, \overline{\Phi}_{I}, B_{I}, B_{I}$ 

each of which has H = 1.

• The representations of these fields:

 $\Phi$ : adjoint, B: bi-fundamental

constrain what gauge-invariant operators can be written down.

[22]

• Some of the H = 1 operators constructed in this way are illustrated as follows:



• For example, on the k = 1 DLCQ ground state we can build the operator:

$$\mathsf{tr}\left(A_1A_2\cdots A_{I-1}\Phi_IA_I\cdots A_{N_2}\right)$$

• Invariance under the orbifold group  $Z_{N_2}$  is achieved by summing over the insertion point. Thus we propose:

$$a_{\Phi,0}^{\dagger}|k=1,m=0
angle\sim\sum_{I=1}^{N_2} \operatorname{tr}\left(A_1A_2\cdots A_{I-1}\Phi_IA_I\cdots A_{N_2}
ight)$$

and similarly for  $\overline{\Phi}, A_I B_I, \overline{B}_I$ .

• Another four are given by:

$$\partial_i \operatorname{tr} (A_1 A_2 \cdots A_{N_2})$$

Thus for k = 1, we have identified the 8 zero-mode bosonic oscillators of the string. For general k, the construction is analogous.

- String oscillators with nonzero mode number can be obtained by inserting phases.
- However, we will see that in our model, this automatically introduces winding states as well.
- In DLCQ string theories, it is well-known that the constraint  $L_0 \overline{L}_0 = 0$  is replaced, in the sector of momentum k and winding m, by:

$$L_0 - \overline{L}_0 = km$$

• This leads to the identification:

$$a_{\Phi,m}^{\dagger}|k=1,m\rangle = \sum_{I=1}^{N_2} \operatorname{tr} \left(A_1 A_2 \cdots A_{I-1} \Phi_I A_I \cdots A_{N_2}\right) \omega^{mI}$$

where

$$\omega = e^{\frac{2\pi i}{N_2}}$$

• The general winding state in the sector of DLCQ momentum 1 is then:

$$\prod_{i=1}^{M} a_{\Phi,n_i}^{\dagger} | k = 1, m \rangle = \sum_{l_M \ge \dots \ge l_2 \ge l_1}^{N_2} \operatorname{tr} \left( A_1 \cdots A_{l_1-1} \Phi_{l_1} \cdots A_{l_i-1} \Phi_{l_i} A_{l_i} \cdots A_{N_2} \right) \omega^{\sum n_i l_i}$$

where the winding number m is defined as the sum of the mode numbers  $n_i$ :

$$m \equiv \sum_{i} n_i.$$

• The gauge theory operators that describe the sector with DLCQ momentum k > 1 are generalizations of the above. For example,

$$a_{\Phi,n}^{\dagger}|k=2,m\rangle = \sum_{I=1}^{2N_2} \operatorname{tr} \left(A_1 \cdots A_{I-1} \Phi_I A_I \cdots A_{N_2} A_1 A_2 \cdots A_{N_2}\right) \omega^{nI}$$

where now

$$\omega = e^{rac{2\pi i}{2N_2}}$$
 and  $m = rac{n}{2}$ 

It is easy to show that the above state vanishes for odd n, an important consistency check.

- [26]
- We see that the DLCQ winding states, represented as gauge theory operators, look like momentum states on the large moose.
- And we already saw that the DLCQ momentum states look like winding states on the large moose.
- This is suggestive of T-duality.

## [27]

#### 4. **Dual:** Non-Relativistic Strings and Membranes

- One can gain more insight into our construction by performing a T-duality over the lightlike DLCQ direction.
- The periodicity of the  $x^-$  direction is a remnant of the combined periodicities in the angles  $\chi$  and  $\phi$ , exhibited earlier.
- Before taking the limit  $N_1, N_2 \rightarrow \infty$ , the periodic direction was space-like. Hence one can perform a T-duality along this direction.
- Let us go back to the original  $AdS_5 \times S^5/Z_{N_2}$  metric and write down only the terms in the t and  $\chi$  directions (i.e., ignoring the transverse space):

$$ds^{2} = R^{2} \left[ -\cosh^{2} \rho \, dt^{2} + \cos^{2} \alpha \cos^{2} \gamma \, d\chi^{2} \right]$$

• Now we make the original replacements for  $\chi, \rho, \alpha, \gamma$  in terms of the ppwave-adapted coordinates:

$$x^{-} = \frac{R^{2}}{2}(t - \chi), \qquad r = \rho R, \qquad x = \alpha R, \qquad y = \gamma R$$

• We do not yet take the limit  $R \to \infty$ , so the metric becomes:

$$ds^{2} = R^{2} \left( \cos^{2} \frac{w}{R} \cos^{2} \frac{y}{R} - \cosh^{2} \frac{r}{R} \right) dt^{2} - 4 \cos^{2} \frac{w}{R} \cos^{2} \frac{y}{R} dt dx^{-} + \frac{4}{R^{2}} \cos^{2} \frac{w}{R} \cos^{2} \frac{y}{R} (dx^{-})^{2}$$

• This procedure has introduced a small  $g_{--}$  in the metric, and we can now T-dualize. We end up with the metric:

$$ds^{2} = -R^{2} \cosh^{2} \frac{r}{R} dt^{2} + \frac{R^{2}}{\cos^{2} \frac{w}{R} \cos^{2} \frac{y}{R}} (dx^{9})^{2}$$

along with a B-field and dilaton:

$$B_{t\,9} = -R^2, \qquad g_s^A = \frac{\sqrt{\alpha' R}}{R_- \cos \frac{w}{R} \cos \frac{y}{R}}$$

Here  $2x^9$  is the T-dual of  $x^-$ .

[29]

• Note that  $x^9$  now has period  $2\pi \frac{\alpha'}{B_{-}}$ , with as before:

$$R_{-} = \frac{\alpha'}{2} g_{\rm YM} \sqrt{\frac{N_1}{N_2}}$$

- There are also Ramond-Ramond fields that we do not write here.
- Evidently some components of the metric, and the *B*-field and string coupling, become infinite as  $R \to \infty$ .
- However, string propagation on this background is finite. The reason is that the *B*-field is a critical electric field and cancels the leading divergent piece in the string world-sheet action:

$$\sqrt{-\det(g)} + B = R^2 \frac{\cosh \frac{r}{R}}{\cos \frac{w}{R} \cos \frac{y}{R}} - R^2 \simeq \frac{1}{2} \sum_{i=1}^8 (x^i)^2 + \mathcal{O}\left(\frac{1}{R^2}\right)$$

• This is just the non-relativistic string propagating in a background with a Newtonian potential of harmonic-oscillator type.

- It is well-known that in a critical electric field with the above scaling, closed strings winding in the direction of the field are light, while the others are are heavy.
- So the non-relativistic closed string (NRCS) that we have arrived at, has only positive windings over the circle.
- The interpretation of our gauge-theory operators winding round the moose is now clear. They are deconstructing these winding states of the NRCS:



- The string of *A*'s had vanishing energy for any winding number, like the light winding strings of NRCS theory.
- The string of  $\overline{A}$ 's winding the other way gives infinitely energetic states as  $N_2 \rightarrow \infty$ , like the NRCS strings winding the wrong way.
- Insertions of phases give the (quantized) momentum states of the NR closed string on  $x^9$ . These can have either sign of momentum.

#### 5. Comments: Parameters and Couplings

(i) Effective 't Hooft coupling:

In the  $\mathcal{N} = 4$  case, this is:

$$\lambda' = \frac{g_{\rm YM}^2 N}{J^2}$$

In the quiver theory we expect:

$$N \to N_1 N_2, \qquad J \to k N_2$$

hence

$$\lambda' = \frac{g_{\rm YM}^2}{k^2} \frac{N_1}{N_2} = \left(\frac{2R_-}{k}\right)^2$$

where we recall that

$$R_{-} \equiv \frac{1}{2} g_{\rm YM} \sqrt{\frac{N_1}{N_2}}$$
 (in  $\alpha' = 1$  units)

(ii) Genus expansion parameter:

In the  $\mathcal{N} = 4$  theory, this is:

$$g_2 = \frac{J^2}{N}$$

In the quiver theory, the corresponding object should be:

$$g_2 = k^2 \, \frac{N_2}{N_1} \sim \frac{N_2}{N_1}$$

This can be checked directly by computing all-genus correlators in the free quiver gauge theory. For example, one finds:

$$\langle 0|\mathrm{tr}\,(A_1A_2\dots A_{N_2})^k\,\mathrm{tr}\,(\overline{A}_{N_2}\overline{A}_{N_2-1}\dots\overline{A}_1)^{k'}|0\rangle = \frac{\delta_{k,k'}}{|x|^{2N_2}}\sum_{l=1}^k \left(\frac{\Gamma(N_1+l)}{\Gamma(N_1+l-k)}\right)^{N_2}$$

As  $N_1, N_2 \rightarrow \infty$  with fixed ratio, this reduces to:

$$\frac{\delta_{k,k'}}{|x|^{2N_2}} \times 2\sum_{l=1}^{\frac{k}{2}} \cosh\left\{ \left(l - \frac{(k+1)}{2}\right) k \frac{N_2}{N_1} \right\}$$

This has an expansion in powers of

$$\left(\frac{N_2}{N_1}\right)^2$$

as expected.

### (iii) Effective coupling:

In the  $\mathcal{N} = 4$  theory there is a combination that describes the effective coupling between states of the same  $\Delta - J$  (at small  $\lambda'$ ):

$$g_{\rm eff} = g_2 \sqrt{\lambda'} = g_{\rm YM} \frac{J}{\sqrt{N}}$$

The corresponding effective coupling in the quiver case would then be:

$$g_{\rm eff} = g_{\rm YM} \, k_{\rm V} \frac{N_{\rm 2}}{N_{\rm 1}}$$

Let us compare this with the type IIA NR closed string coupling, given by the familiar (NCOS) formula:

$$g_{\mathrm{NR}} = g_{\mathrm{NCOS}}^2 = g_s^A \sqrt{\frac{\det(g+B)}{\det g}}$$

If we evaluate this on our background, we find a spatially varying coupling:

$$g_{\rm NR} = \frac{g_s^B}{R_-} \sqrt{\sum_{i=1}^8 (x^i)^2}$$

• Since the string is trapped in a harmonic potential, the states are confined to a finite region, and the coupling constant will reduce to:

$$g_{\rm NR} \sim \frac{g_s^B}{R_-} \sim g_{\rm YM} \sqrt{\frac{N_2}{N_1}} = g_{\rm eff}$$

We see that the effective coupling among a subclass of gauge theory states can be identified with the effective coupling between the winding NR closed strings.

• By taking this coupling to be large, we go over to M-theory . The wound NR string becomes a non-relativistic membrane wound over the  $x^9, x^{10}$  directions.

(iv) pp-wave mass parameter  $\mu$ :

In the noncompact (usual) pp-wave background, the parameter  $\mu$  can be scaled to any value by rescaling

$$x^+ \to \Lambda x^+, \qquad x^- \to \Lambda^{-1} x^-$$

for some  $\Lambda$ .

It is also true that in a flat-space DLCQ background, the DLCQ radius can be scaled to any value (by the same procedure).

But in the DLCQ pp-wave, things are different. We can simultaneously change  $\mu$  and the DLCQ radius  $R_{-}$  by the above scaling, but not either one separately. This is why there is one physically relevant parameter.

# (v) Deconstruction:

The same quiver gauge theory that we have been discussing was studied last year in a different limit:  $N_2 \rightarrow \infty$ ,  $N_1$  fixed, and  $\alpha' \rightarrow 0$ .

It was proposed that in this limit, taken along the Higgs branch, two additional dimensions are dynamically deconstructed and one ends up with the (2,0) field theory. This arises as the decoupled theory on M5-branes.

We take  $N_1, N_2 \rightarrow \infty$  together with the near-horizon limit of the D3-branes. Also, we zoom in on a lightlike geodesic. This is similar to being in the Higgs branch, because we miss the orbifold singularity.

Our final result is a DLCQ pp-wave of radius  $\sim \sqrt{N_1/N_2}$ , or a Galilean string/membrane wrapped on a circle/torus of radius  $\sim \sqrt{N_2/N_1}$ .

It is not clear if conventional deconstruction can be obtained from this as  $N_2/N_1 \rightarrow \infty$ .

# 6. Conclusions

- Large quiver theories give a nontrivial generalization of the gauge/pp-wave correspondence. They describe the Discrete Light Cone Quantization of type IIB string theory on a pp-wave background.
- They also deconstruct the non-relativistic closed string/membrane. The operator wrapping the moose once is a DLCQ string bit.
- It is clearly of interest to study string interactions in this model. For small values of k, the formulae should be simpler and hopefully we can check more.
- Another open problem is to directly study the non-relativistic string in a potential and compare it with gauge theory.
- But there is a more important conceptual question: DLCQ is often associated to a fundamental formulation of a theory (as with M(atrix) Theory). Is there such a fundamental formulation – M(oose) Theory – hidden here?



# THE END