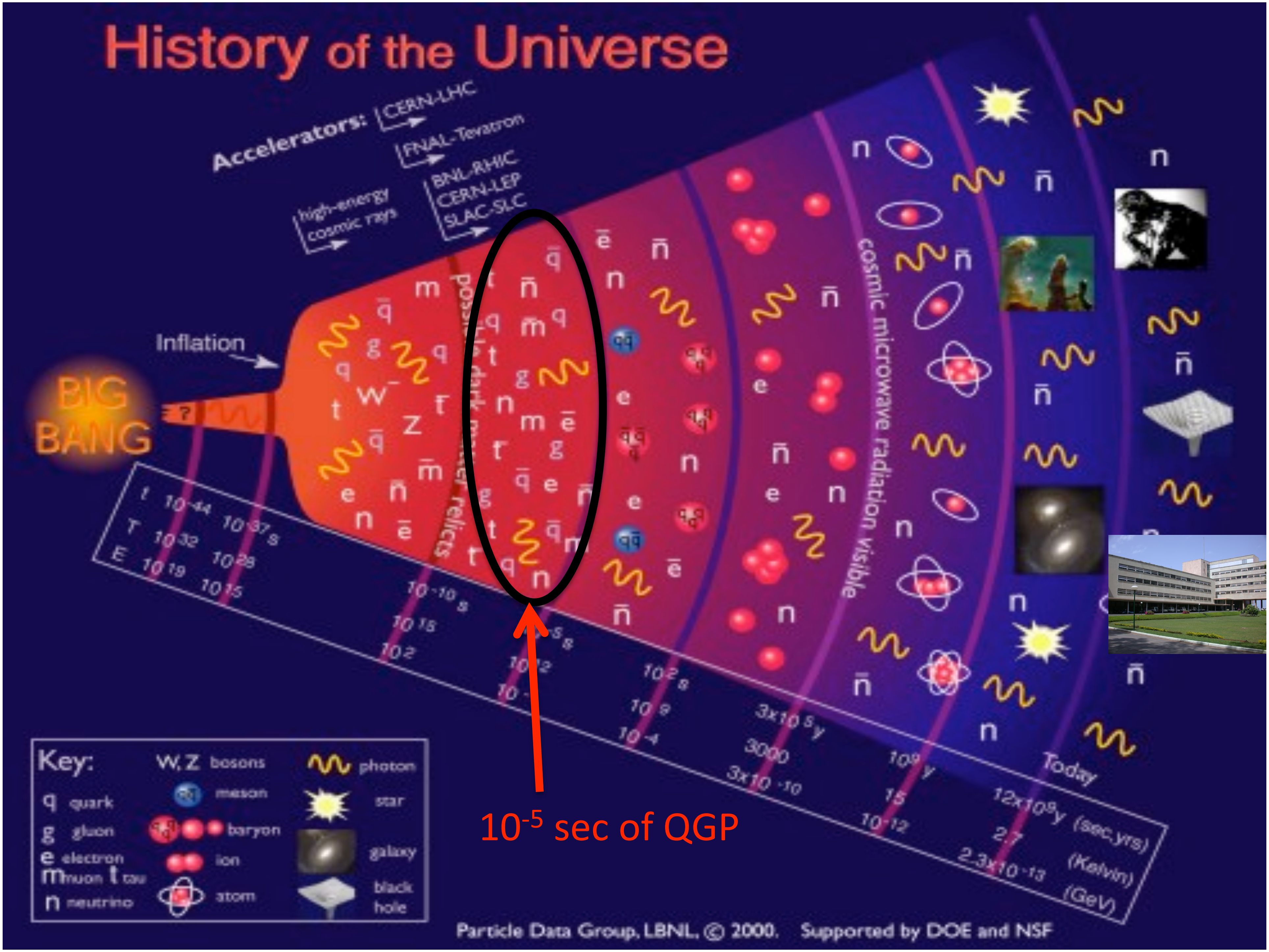
The QCD equation of state and transition at zero chemical potential

Michael Cheng

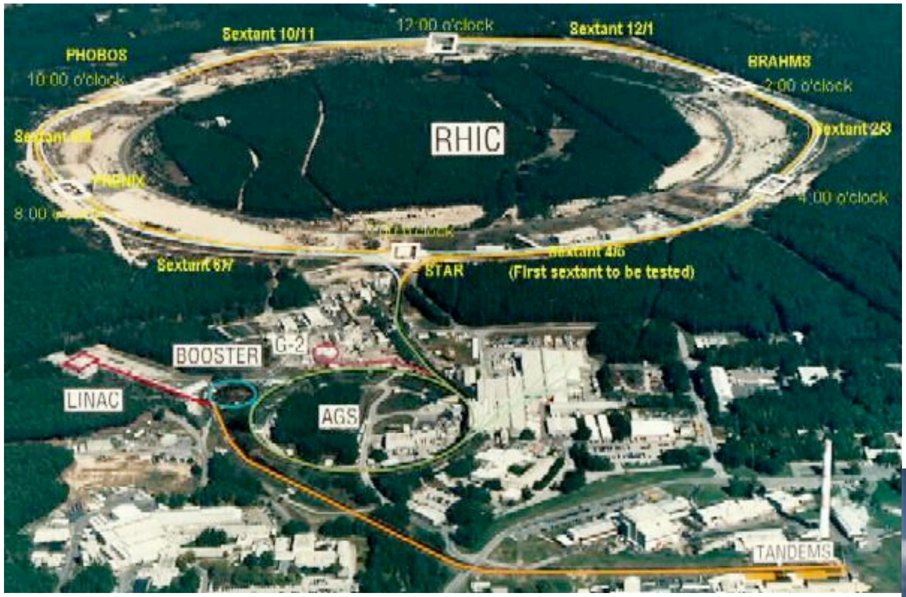
Lawrence Livermore National Laboratory

Strong Interaction in the 21st Century
February 11, 2010
Tata Institute of Fundamental Research, Mumbai



10^{-5} sec of QGP

Heavy Ion Colliders



RHIC

LHC

A Heavy Ion Collision

Figure from U. Muenster

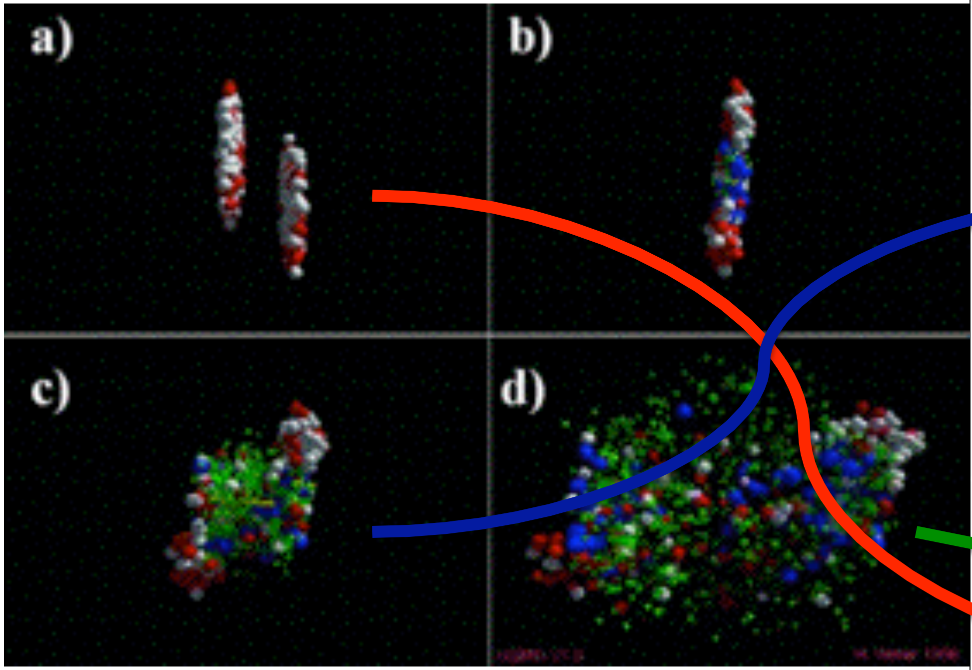
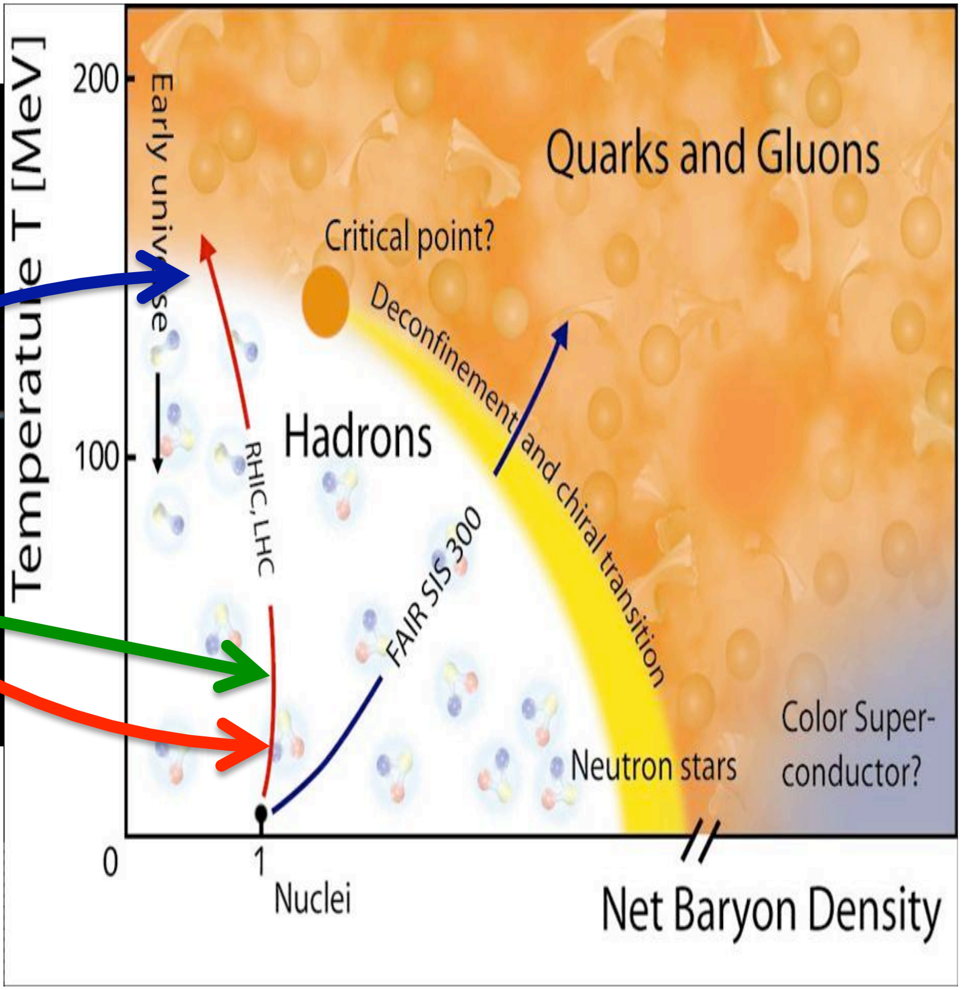


Figure from GSI



QGP lifetime: 10^{-23} sec.

Overview

Calculation of the bulk thermodynamics of QCD matter at finite temperature but zero density.

Examination of observables that signal deconfinement and chiral symmetry restoration.

High-temperature improved staggered fermions (p4 and asqtad) at physical quark masses.

Results with N_t=6,8 (HotQCD and RBC-Bielefeld)
Results at N_t=12 (HotQCD Preliminary)

The HotQCD and RBC-Bielefeld collaborations

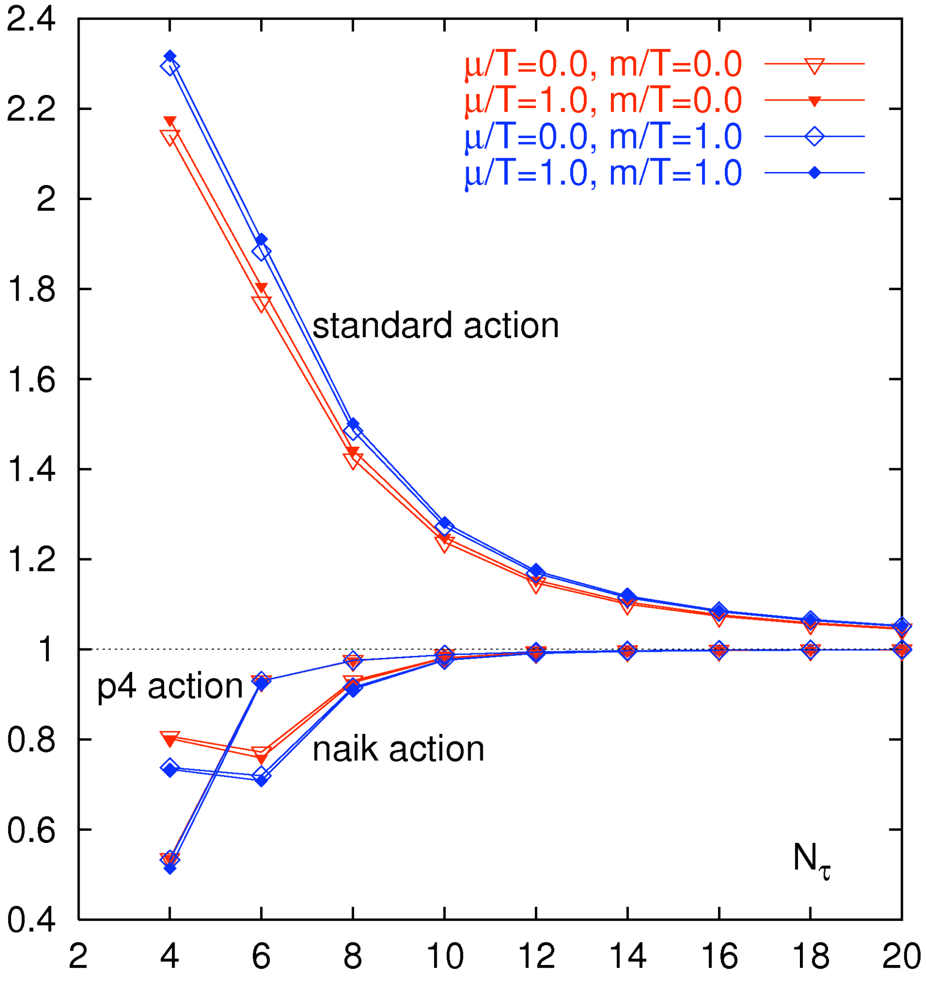
- T. Battacharya (LANL)
- A. Bazavov (Arizona)
- M. Cheng (LLNL)
- N. Christ (Columbia)
- C. DeTar (Utah)
- S. Ejiri (BNL)
- S. Gottlieb (Indiana)
- R. Gupta (LANL)
- U. Heller (APS)
- P. Hegde (BNL)
- C. Jung (BNL)
- O. Kaczmarek (Bielefeld)
- F. Karsch (BNL/Bielefeld)
- E. Laermann (Bielefeld)
- L. Levkova (Utah)
- R. Mawhinney (Columbia)
- C. Miao (BNL)
- S. Mukherjee (BNL)
- P. Petreczky (BNL)
- D. Renfrew (Columbia)
- C. Schmidt (FIAS/GSI)
- R. Soltz (LLNL)
- W. Soeldner (GSI)
- R. Sugar (UCSB)
- D. Toussaint (Arizona)
- W. Unger (Bielefeld)
- P. Vranas (LLNL)

Computational resources from LLNL, USQCD, NYCCS, Juelich

Overview of the Calculations

- Equation of State and transition region with asqtad and p4 fermions at Nt = 6, 8. (HotQCD: Bazavov, *et. al.,* Phys.Rev.D80:014504,2009)
  - m_s approximately physical, $m_{ud} = 0.1\ m_s$ -> $m_\pi = 220, 260$ MeV.
  - 32^3x8 and 32^3x6, 24^3x6 finite T volumes, 32^4 T=0 volume.
  - 140 MeV < T < 540 MeV
- RBC-Bielefeld Collaboration: N_t=8 p4 fermions with “physical quark masses (arXiv:0911.2215)
  - $m_{ud} = 0.05\ m_s$ -> > $m_\pi = 150$ MeV
  - 140 MeV < T < 260 MeV
- **Preliminary HotQCD**: $N_t = 12$ with asqtad fermions
  - “Physical” quark masses $m_{ud} = 0.05\ m_s$.
  - 140 MeV < T < 200 MeV

High Temperature Improvement



Allton, *et. al.*, Phys.Rev. D68 (2003) 014507

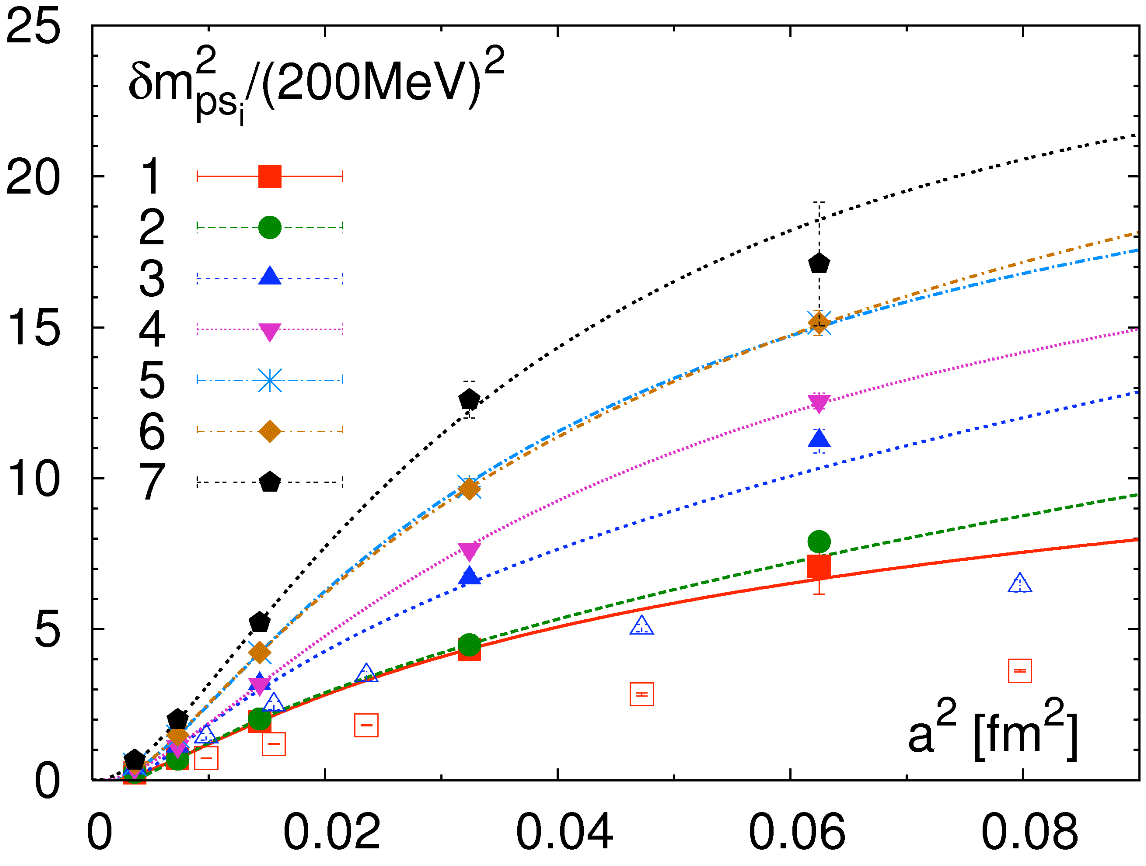
Nearest neighbor terms in Dirac operator augemented with three-link terms.

Removes O(a^2) effects in quark dispersion relation -> controls thermodynamics in high T limit.

Asqtad action developed for good scaling in T=0 sector.

Compare to “unimproved” staggered.

Flavor Symmetry Breaking



P4, asqtad employ “fat-link” smearing, but do not do a great job of supressing flavor symmetry breaking.

T = 180 MeV:

$N_t = 6 \rightarrow a^2 \approx 0.033$ fm^2
$N_t = 8 \rightarrow a^2 \approx 0.019$ fm^2

Stout and HISQ have better flavor symmetry.

Courtesy of P. Petreczky

Equation of State

Calculating EoS

Use integral method. Calculate ε-3p, aka “interaction measure” or “conformal anomaly”

When temperature is only relevant energy scale, ε - 3p = 0 – true for massless ideal gas, conformal theories, QGP at very high temperatures.

$$\frac{\epsilon - 3p}{T^4} = T\frac{\partial}{\partial T}\left(\frac{p}{T^4}\right) = \frac{\Theta^{\mu\mu}(T)}{T^4} = \boxed{\frac{\Theta^{\mu\mu}_G(T)}{T^4}} + \boxed{\frac{\Theta^{\mu\mu}_F(T)}{T^4}};$$

$$\boxed{\frac{\Theta^{\mu\mu}_G(T)}{T^4} = -N_t^4\left(\frac{d\beta}{d\ln a}\right)\left[\langle s_G\rangle_0 - \langle s_G\rangle_T\right]}$$

$$\boxed{\frac{\Theta^{\mu\mu}_F(T)}{T^4} = N_t^4\left(\frac{d\beta}{d\ln a}\right)\left(\frac{dm_{ud}}{d\beta}\right)\left[2\left(\langle\bar{\psi}\psi\rangle_{l,0} - \langle\bar{\psi}\psi\rangle_{l,T}\right) + \frac{m_s}{m_{ud}}\left(\langle\bar{\psi}\psi\rangle_{s,0} - \langle\bar{\psi}\psi\rangle_{s,T}\right)\right.}$$

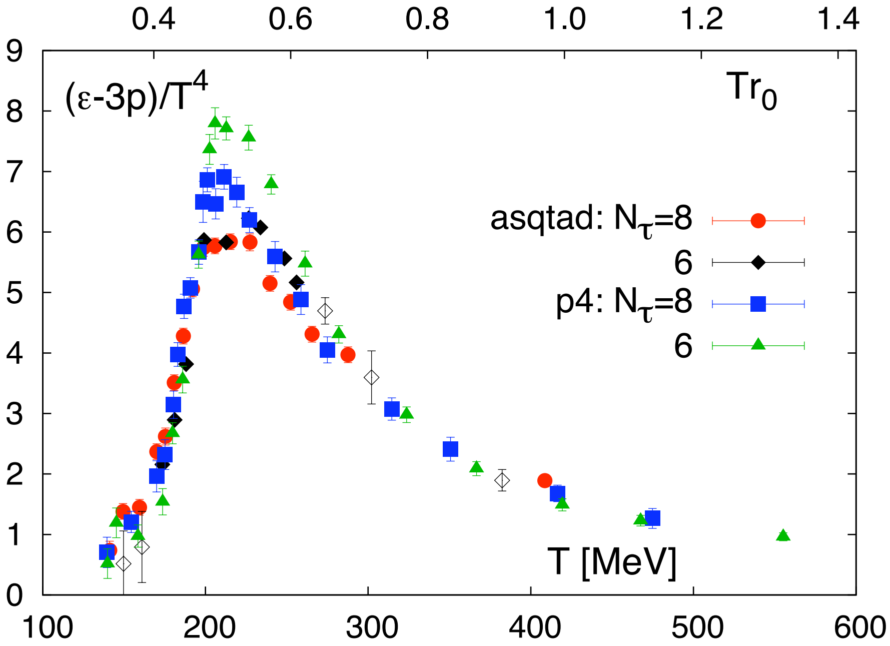
Calculating EoS

Calculate pressure by integrating “interaction measure from the low temperature phase, T_0.

Energy density, entropy density, and speed of sound then is easily calculated via their thermodynamic definitions.

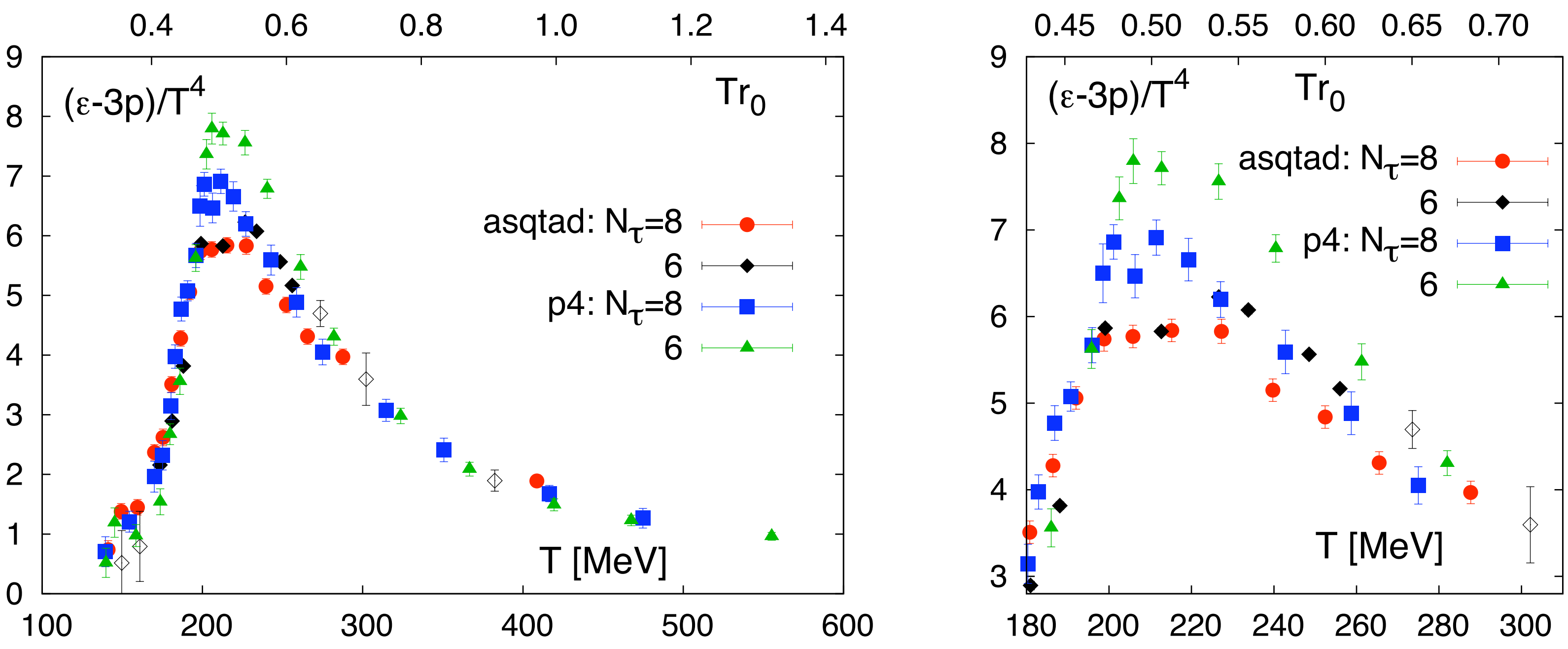
$$\begin{aligned}
\frac{\epsilon - 3p}{T^4} &= T\frac{\partial}{\partial T}\left(\frac{p}{T^4}\right) = \frac{\Theta^{\mu\mu}(T)}{T^4} = \frac{\Theta^{\mu\mu}_G(T)}{T^4} + \frac{\Theta^{\mu\mu}_F(T)}{T^4}; \\
\frac{\Theta^{\mu\mu}_G(T)}{T^4} &= -N_t^4\left(\frac{d\beta}{d\ln a}\right)\left[\langle s_G\rangle_0 - \langle s_G\rangle_T\right] \\
\frac{\Theta^{\mu\mu}_F(T)}{T^4} &= N_t^4\left(\frac{d\beta}{d\ln a}\right)\left(\frac{dm_{ud}}{d\beta}\right)\left[2\left(\left\langle\bar\psi\psi\right\rangle_{l,0} - \left\langle\bar\psi\psi\right\rangle_{l,T}\right) + \frac{m_s}{m_{ud}}\left(\left\langle\bar\psi\psi\right\rangle_{s,0} - \left\langle\bar\psi\psi\right\rangle_{s,T}\right)\right]
\end{aligned}$$

$$\frac{p}{T^4} = \int_{T_0}^{T} dT'\,\frac{\Theta^{\mu\mu}(T')}{T'^5}; \quad \frac{\epsilon}{T^4} = \frac{\Theta^{\mu\mu}(T) + 3p}{T^4}; \quad \frac{s}{T^3} = \frac{\epsilon + p}{T^4}; \quad c_s^2 = \frac{dp}{d\epsilon}$$



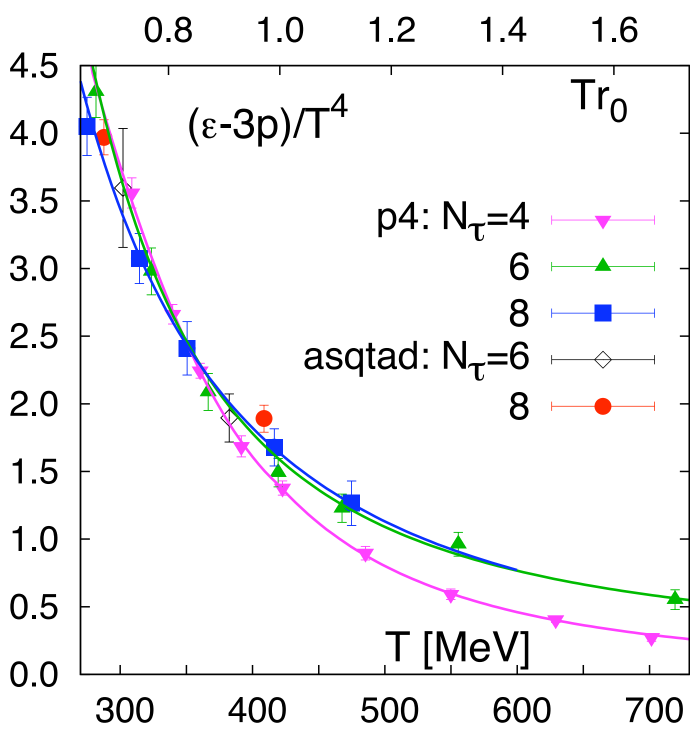
Bazavov, *et. al.,* Phys.Rev.D80:014504,2009

- Both asqtad and p4 actions reveal same qualitative features for the interaction measure – rapid increase from low T regime with peak just above transition region, followed by rapid drop-off in the high temperature region.



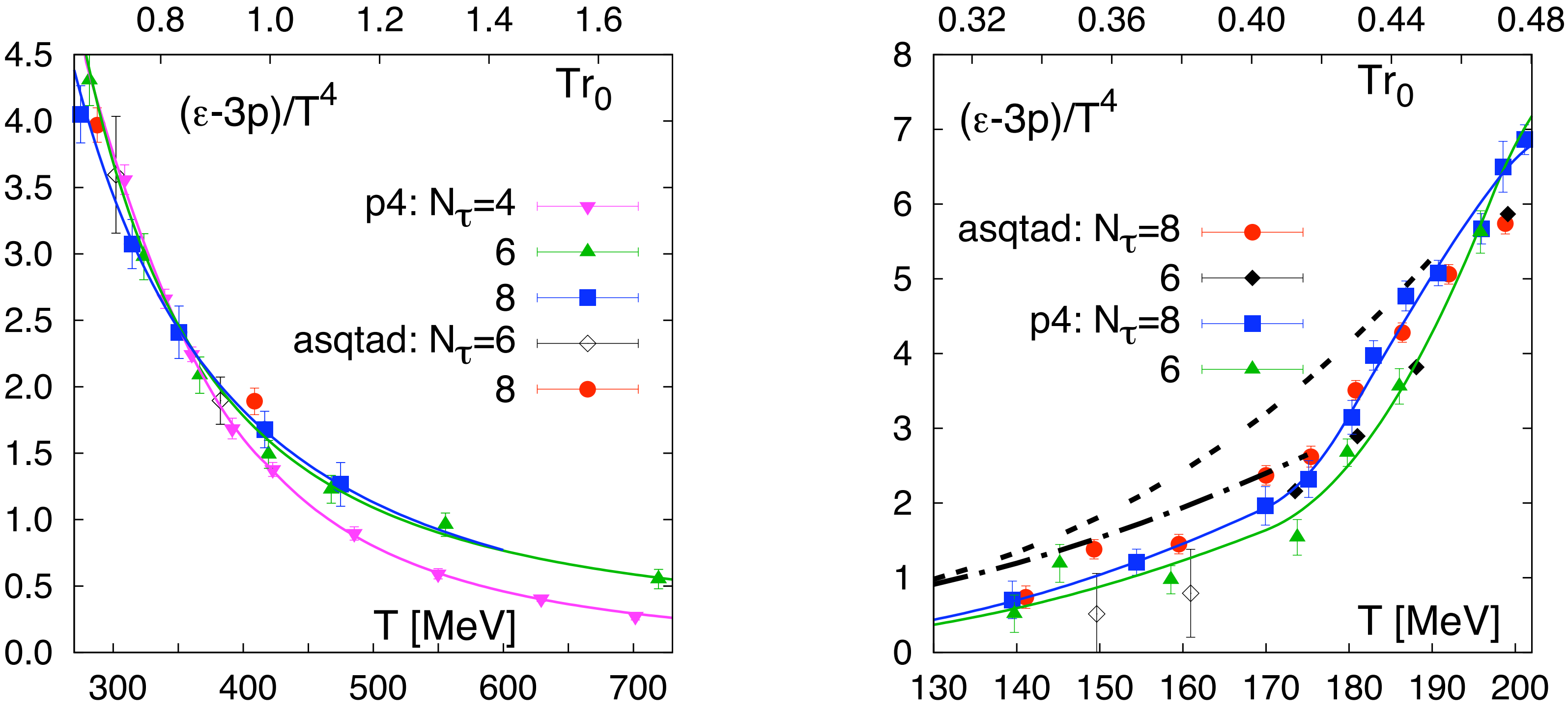
Bazavov, *et. al.,* Phys.Rev.D80:014504,2009

- Largest differences in the vicinity of the peak.
- Scaling errors appear to be smaller for asqtad action compared to p4.
- Peak height is 15% smaller for asqtad action



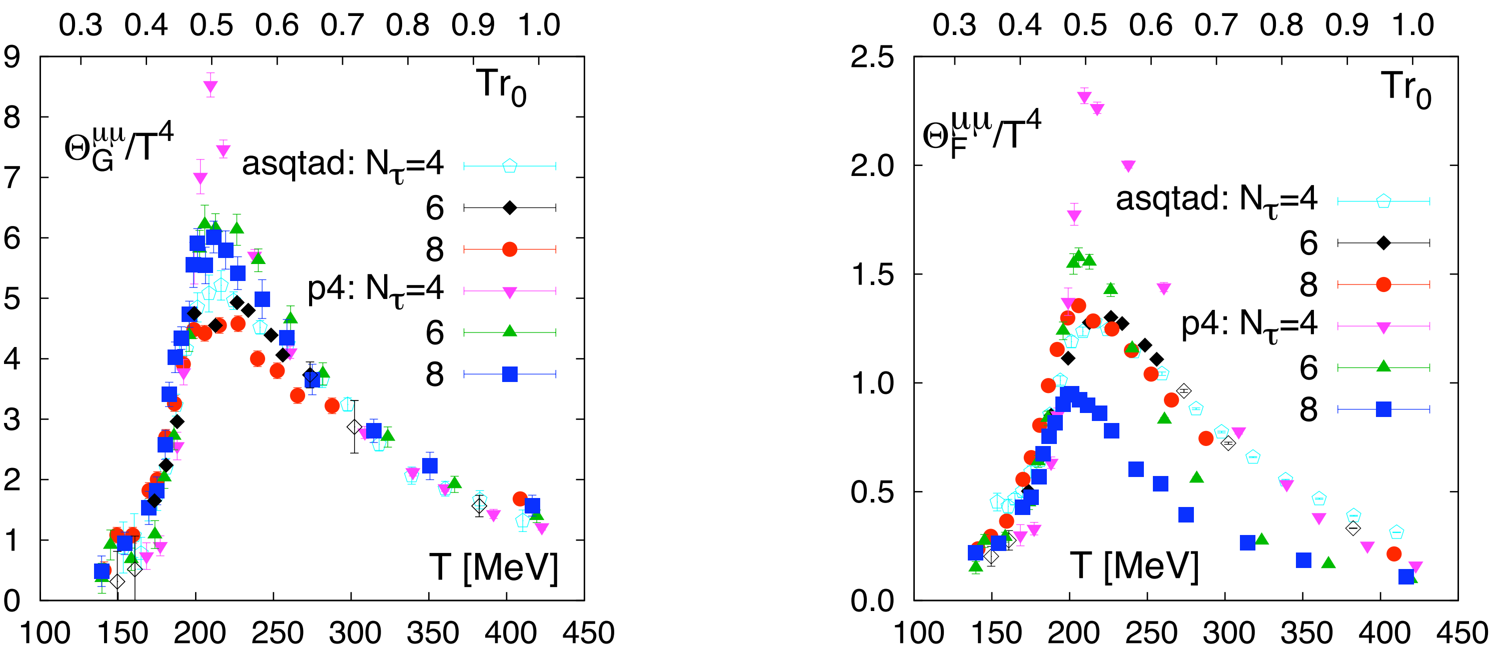
Bazavov, *et. al.,* Phys.Rev.D80:014504,2009

- Smallest scaling errors at high temperature. N_t = 6, 8 coincide for both p4 and asqtad.
- Deviation from N_t = 4 results.



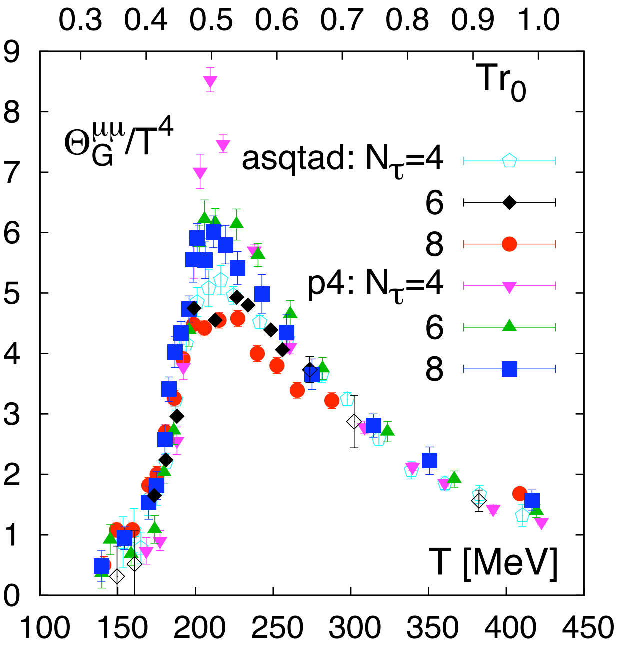
Bazavov, *et. al.,* Phys.Rev.D80:014504,2009

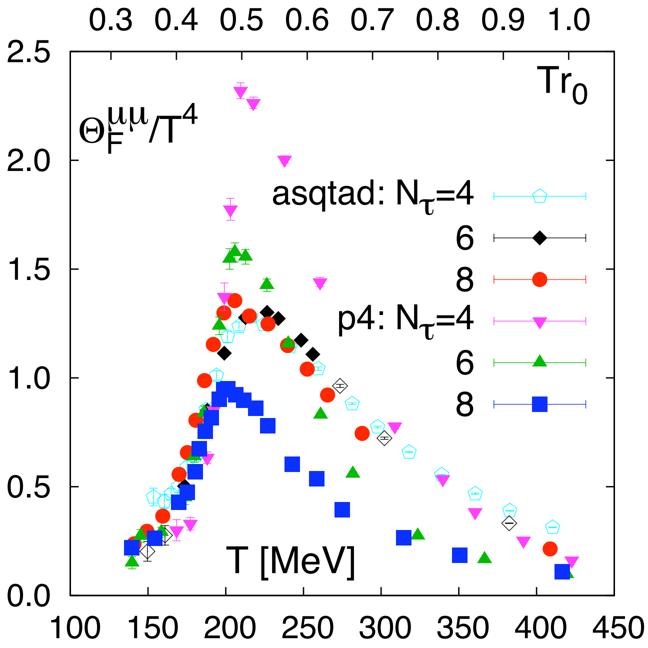
- Larger cut-off effects at low temperature – largest lattice spacings
- Approx. 5 MeV shift of the entire curve going from N_t=6 to N_t=8.
- Comparison to HRG also shown (dashed lines) for resonance cut off m = 1.5, 2.5 GeV. Lattice data lie below HRG results.
- Expect this temperature regime to be hadron-dominated – hadron masses are heavier than physical.
- See also P. Petreczky and P. Huovinen arXiv:0912.2541



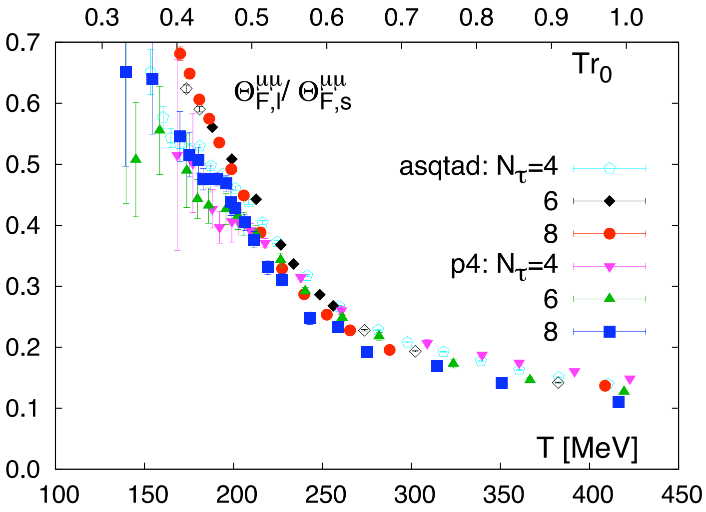
- Contributions from gluonic and fermionic operators in the interaction measure.
- Fermionic operator contributes only about 15% of total interaction measure
- Most of the fermionic effect bound up in interactions with the gauge field.

Bazavov, *et. al.,* Phys.Rev.D80:014504,2009

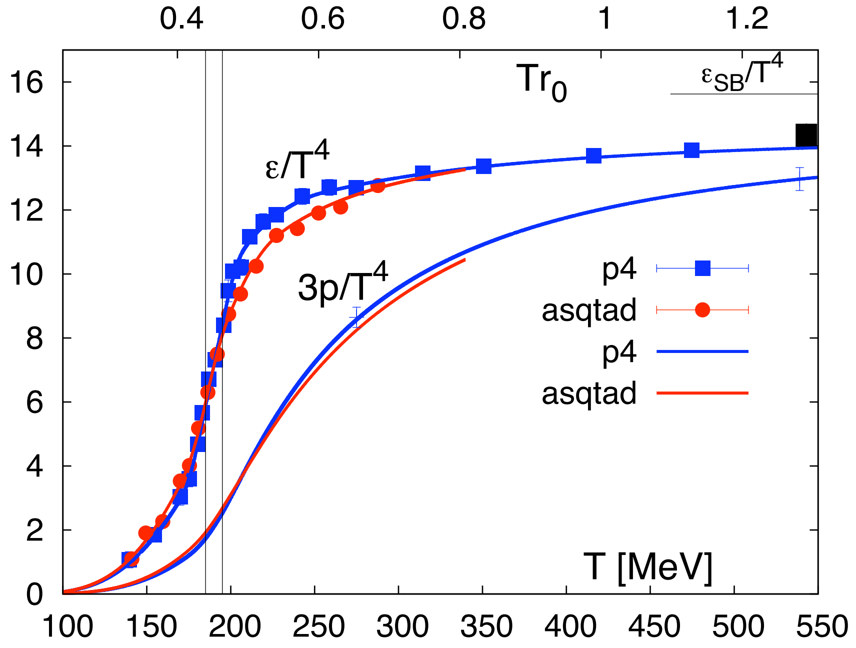




- Much of scaling error comes from dm/dβ. When this contribution is divided out, asqtad and p4 have better agreement.
- Also note that “fermionic” part of interaction measure has larger contributions from light quark part near peak.

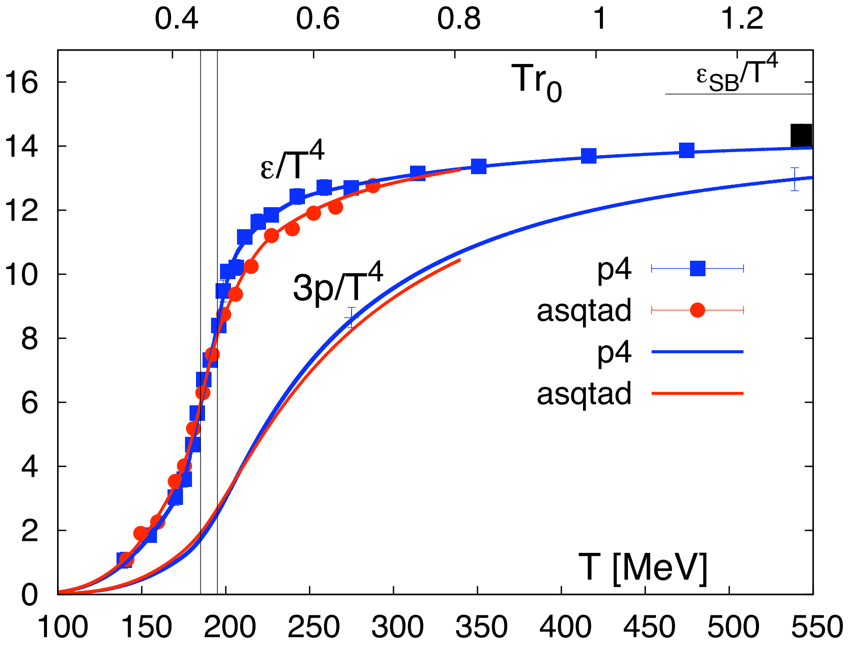


Bazavov, *et. al.,* Phys.Rev.D80:014504,2009

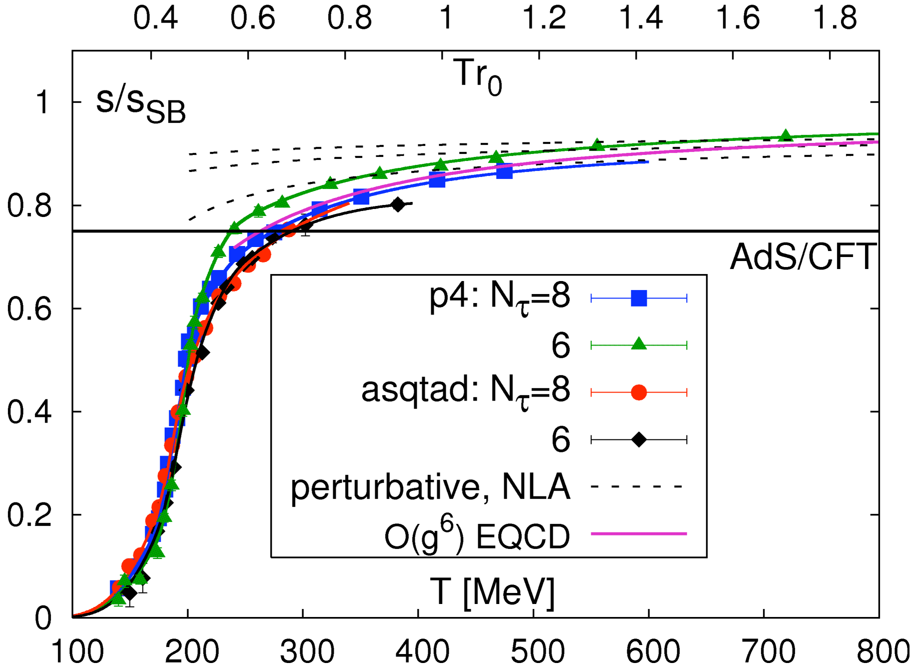


Bazavov, *et. al.,* Phys.Rev.D80:014504,2009

- All observables rise rapidly in the transition region, 185 MeV < T < 195 MeV.
- Systematic error in the choice of lower integration limit, T_0: Set T_0=100 MeV or linear interpolation to T_0=0. Error indicated by bars on the pressure curve.
- Also assume that p = 0 at lower limit of integration: T_0=100 MeV. Systematic upward shift by p ≠ 0 at T_0 = 100 MeV calculated from HRG.
- Differences between p4 and asqtad reflect differences in interaction measure. 5% difference for T > 230 MeV, becoming about 10% at T = 200 MeV.
- Small scaling errors in p4 – about 5% shift between N_t=6 and N_t=8
- No significant scaling errors in asqtad.



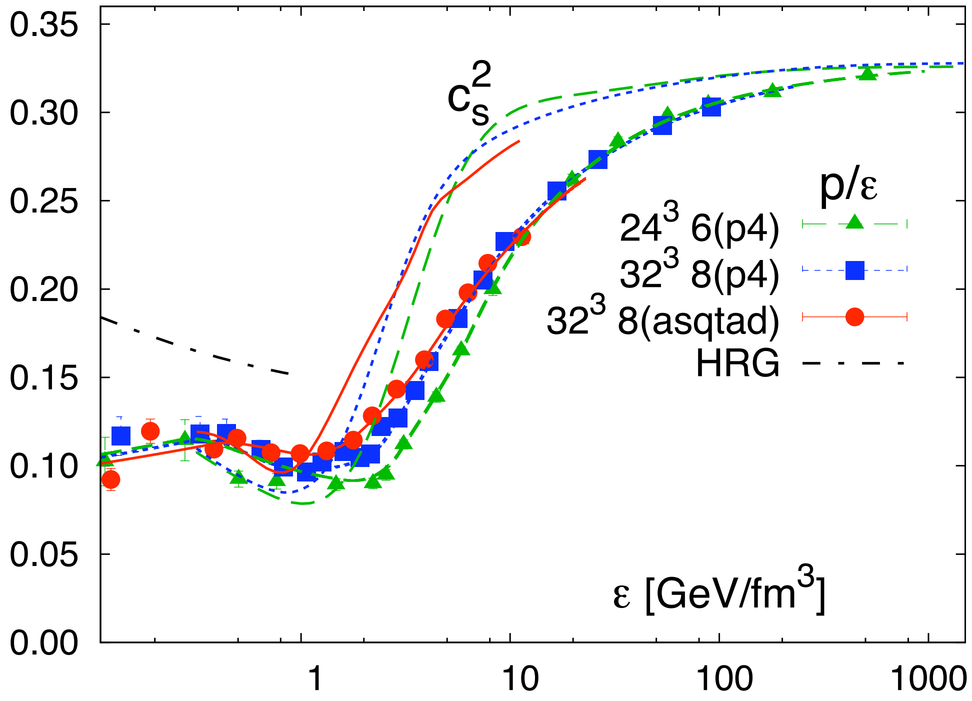
Bazavov, *et. al.*, Phys.Rev.D80:014504,2009



MC, *et. al.*, arXiv: 0911.2215

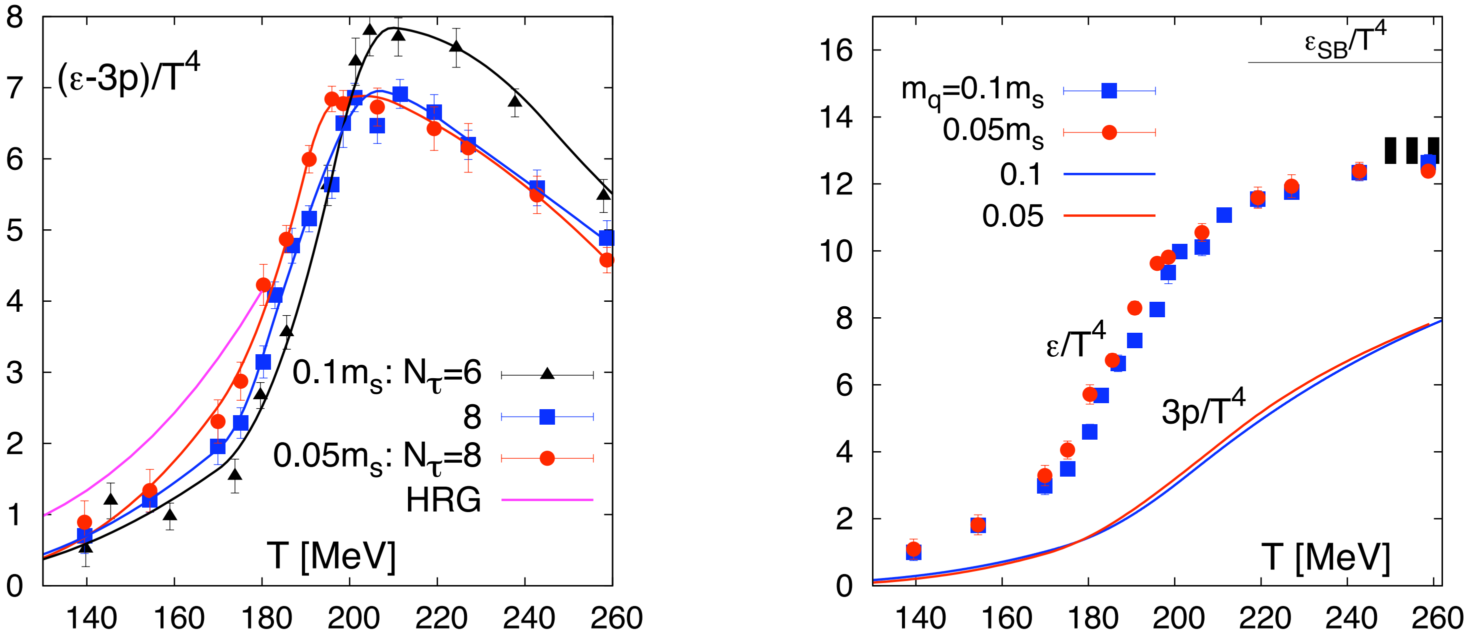
- Entropy density $s/T^4 = (\varepsilon+p)/T^4$
- Compare with perturbative calculations and AdS/CFT

Bazavov, *et. al.*,
Phys.Rev.D80:0145
04,2009



- Enough data points to allow a smooth parameterizations of p(T) and ε(T), from which we can calculate the speed of sound.
- c_s^2 saturates the free-field value $c_s^2 = 1/3$ rather quickly.
- Minimum in c_s near the transition region, the place where the QCD medium is softest, when ε ~ 1 GeV/fm^3
- Poor agreement with HRG result at low temperature – expected because quark masses are too heavy, and c_s becomes sensitive to small errors in p(T) and ε(T) as well as their parameterizations.

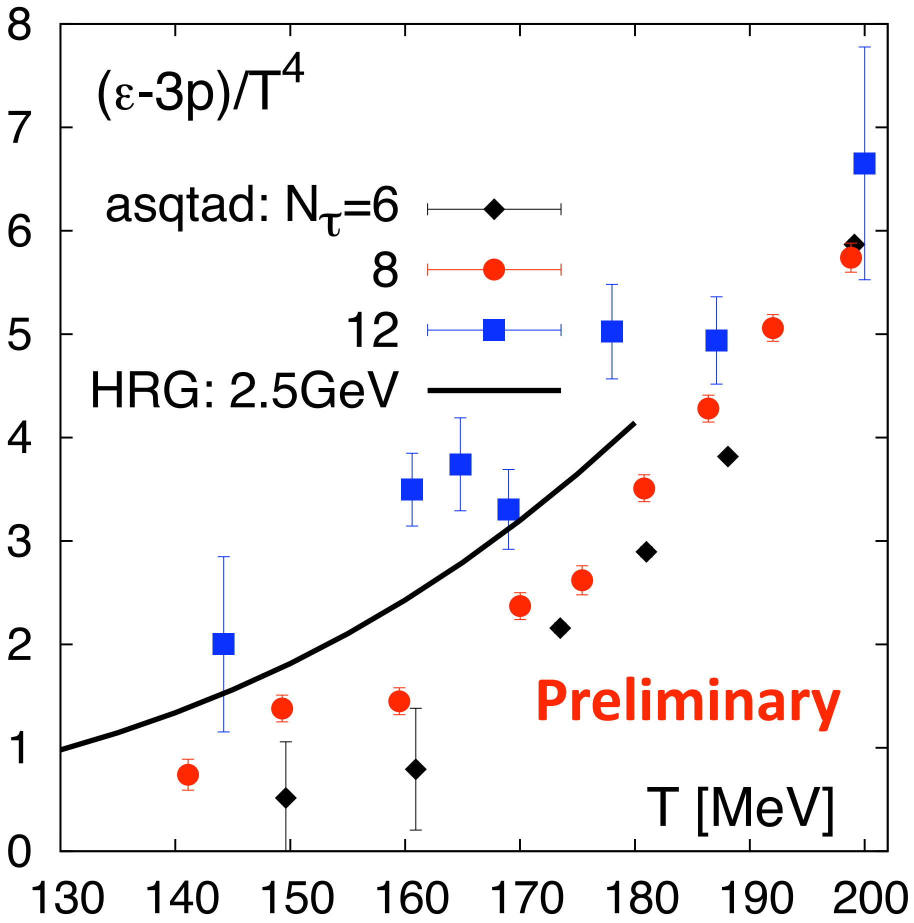
$m_{ud} = 0.05\ m_s$



MC, *et. al.*, arXiv: 0911.2215

- “Physical” quark mass enhance interaction measure at fixed T, relative to heavier quark mass -> hadron masses closer to their actual values.
- Not much effect on interaction measure for T > 200 MeV -> quark masses no longer play much role after hadrons dissipate.

N_t = 12

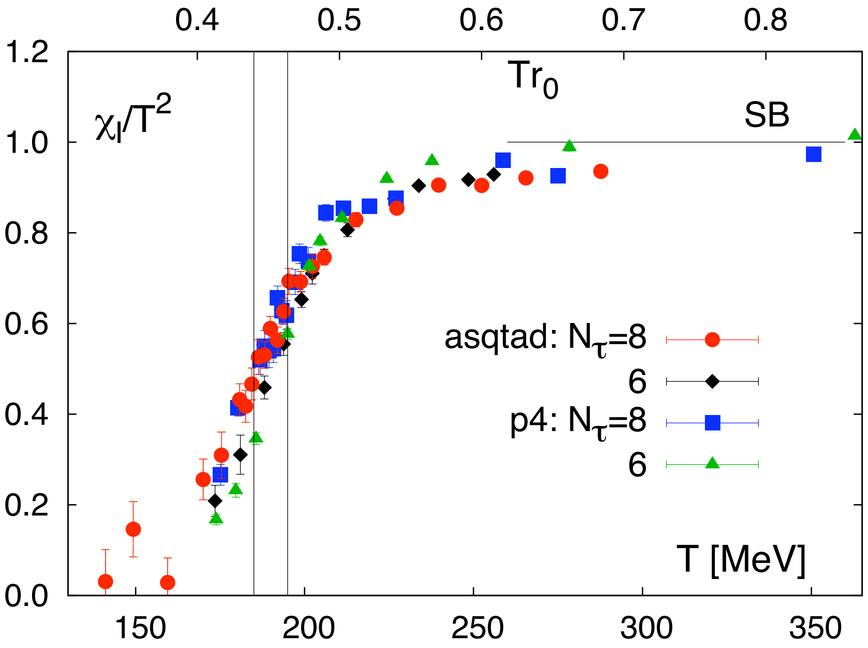


- N_t = 12 data shifts ε – 3p upwards compared to N_t = 6, 8
- Several effects:
  - Smaller lattice spacing shifts curve leftward.
  - Smaller quark mass also shifts to smaller T.
  - Reduced flavor symmetry breaking in hadron spectrum lifts ε – 3p.
- Better agreement now with HRG gas model.

Transition

Deconfinement vs. Chiral Transition

- Two distinct transitions with different order parameters
- Deconfinement:
  - Quarks and gluons are liberated from hadronic bound states
  - Probed by calculating Polykov loop and quark number susceptibilities
- Chiral symmetry restoration:
  - Vacuum chiral condensate $\langle\bar{\psi}\psi\rangle \neq 0$ "melted" at high temperature into a phase with chiral symmetry $\langle\bar{\psi}\psi\rangle = 0$
  - Probed by calculating chiral condensate, chiral susceptibility
- Results from Aoki, *et. al.* (hep-lat/0609068, arXiv:0903.4155) give T_c ≈ 150 MeV for chiral symmetry and T_c ≈ 175 MeV
- Contrast with earlier RBC-Bielefeld results T_c ≈ 190 MeV without for both deconfinement and chiral.

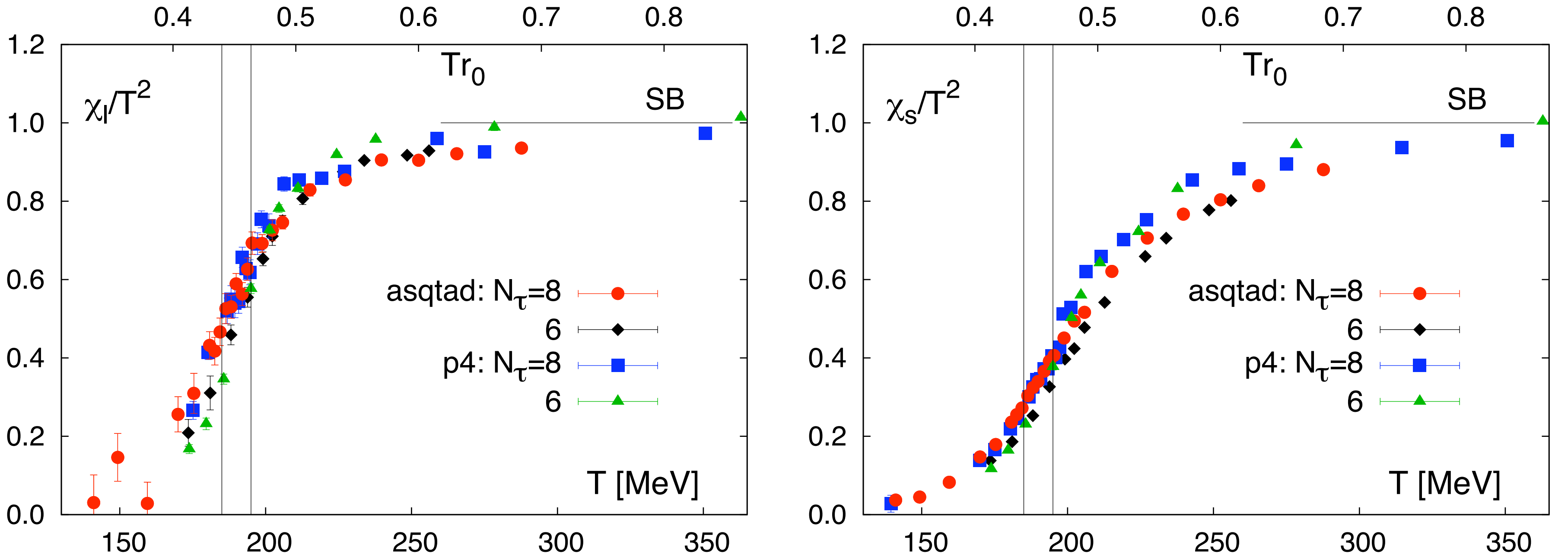


Bazavov, *et. al.,* Phys.Rev.D80:014504,2009

Quark Number susceptibility measures fluctuations in the degrees of freedom that carry net quark number, *i.e.,* hadrons at low temperature, quarks at high temperature.

$$\frac{\chi_q}{T^2} = \frac{1}{VT^3}\frac{\partial^2 \ln Z}{\partial\left(\mu_q/T\right)^2}$$

Both light and strange susceptibilities rise most rapidly in the region (185 MeV < T< 195 MeV) and quickly approach free-field ideal gas value $\chi_q/T^2 = 1$.

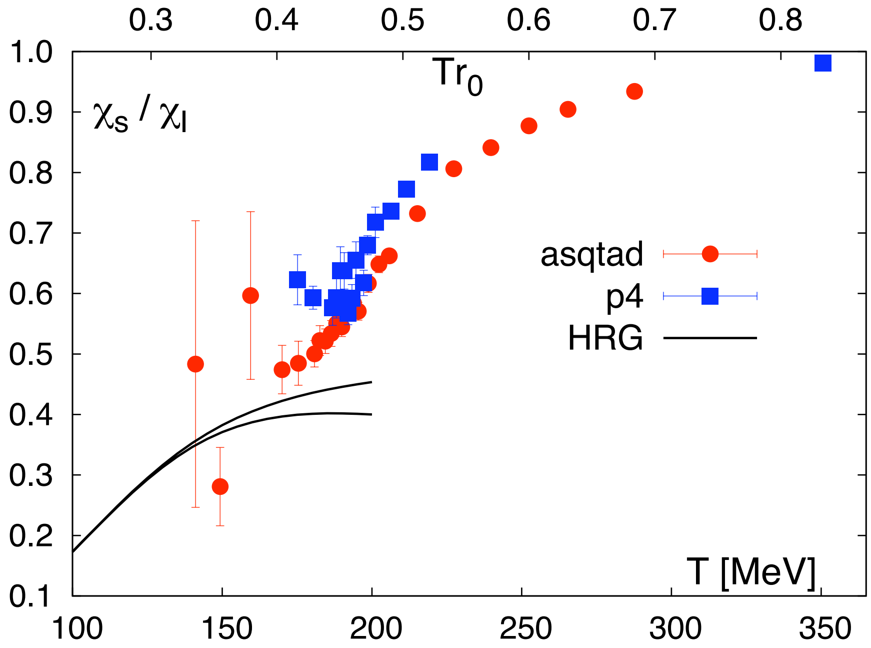


Bazavov, *et. al.,* Phys.Rev.D80:014504,2009

Quark Number susceptibility measures fluctuations in the degrees of freedom that carry net quark number, *i.e.,* hadrons at low temperature, quarks at high temperature.

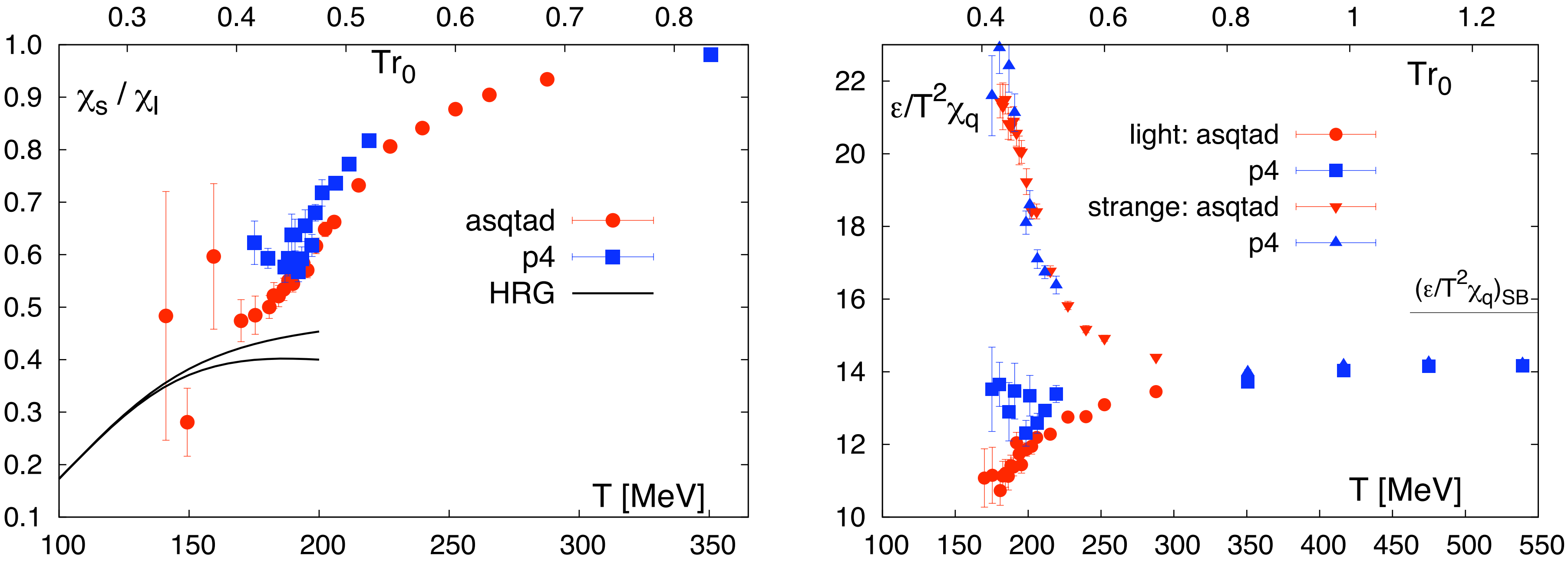
$$\frac{\chi_q}{T^2} = \frac{1}{VT^3} \frac{\partial^2 \ln Z}{\partial \left(\mu_q/T\right)^2}$$

Both light and strange susceptibilities rise most rapidly in the region (185 MeV < T< 195 MeV) and quickly approach free-field ideal gas value $\chi_q/T^2 = 1$.



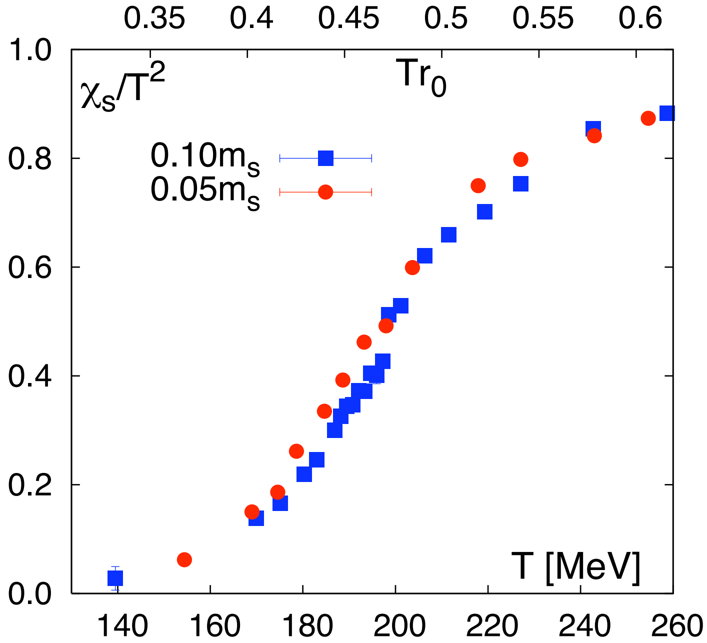
Bazavov, *et. al.,* Phys.Rev.D80:014504,2009

- χ_l rises more quickly - directly sensitive to the lightest hadronic modes at low temperature, the pions, while $\chi_s \sim \exp(-m_K/T)$ at low temperature.
- $\chi_{s}/\chi_l \sim 1$ at high temperature, but is approximately 0.5 below the transition, consistent with HRG calculation.



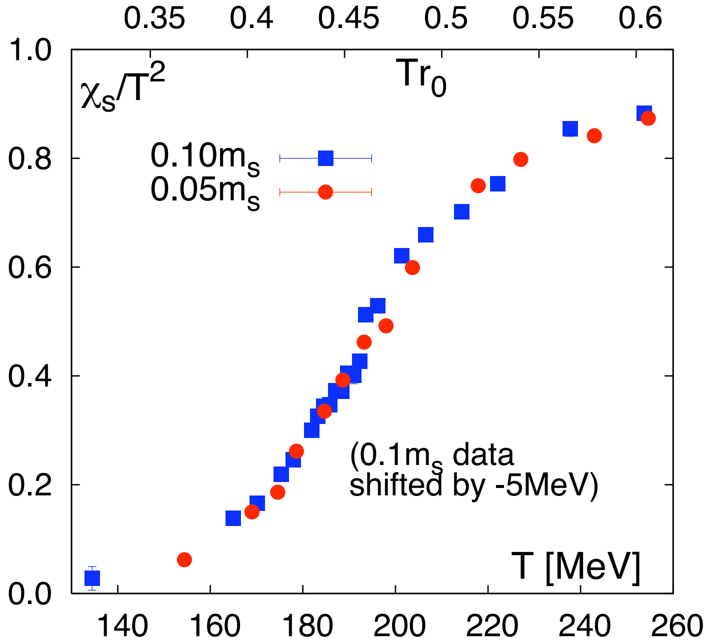
Bazavov, *et. al.,* Phys.Rev.D80:014504,2009

- χ_l tracks energy density - $\varepsilon/(T^2 \chi_l)$ is almost constant in high temperature regime T > 300 MeV. Fluctuation in light quark degrees of freedom reflect liberation of degrees of freedom in energy density.
- Meanwhile, $\varepsilon/(T^2 \chi_s)$ diverges at low temperature as strange quark number susceptibility is more suppressed at low temperature.



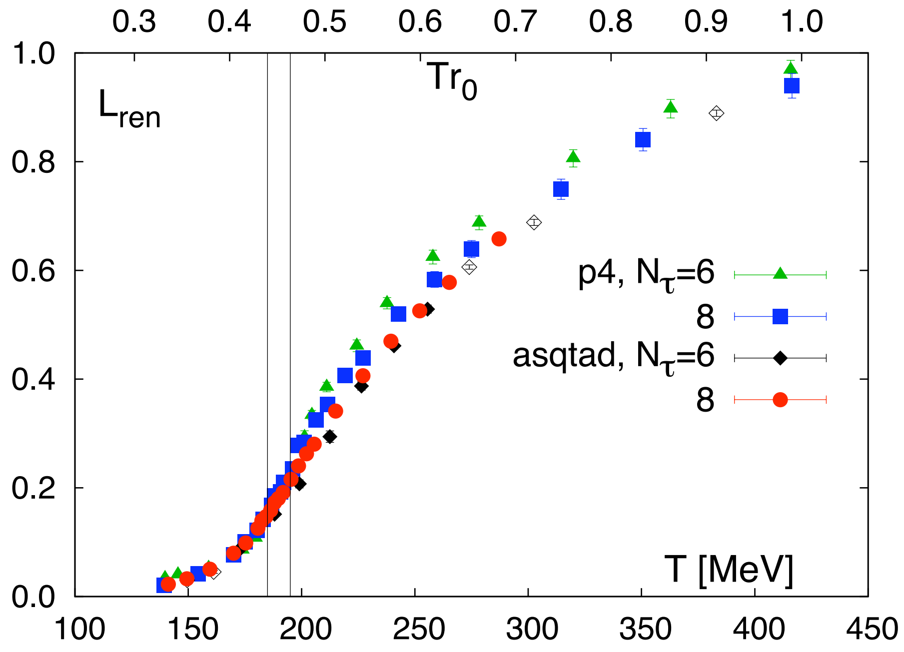
MC, *et. al.*, arXiv: 0911.2215

- Results for p4 action for N_t=8 with $m_{ud} = 0.05\ m_s$
- Extrapolation from results at $m_{ud} = 0.20\ m_s$ and $m_{ud} = 0.10\ m_s$ imply expected 5 MeV downward shift of transition with decreased mass.
- Results confirm this expectation for T < 200 MeV, but mass dependence perhaps less drastic for T > 200 MeV.



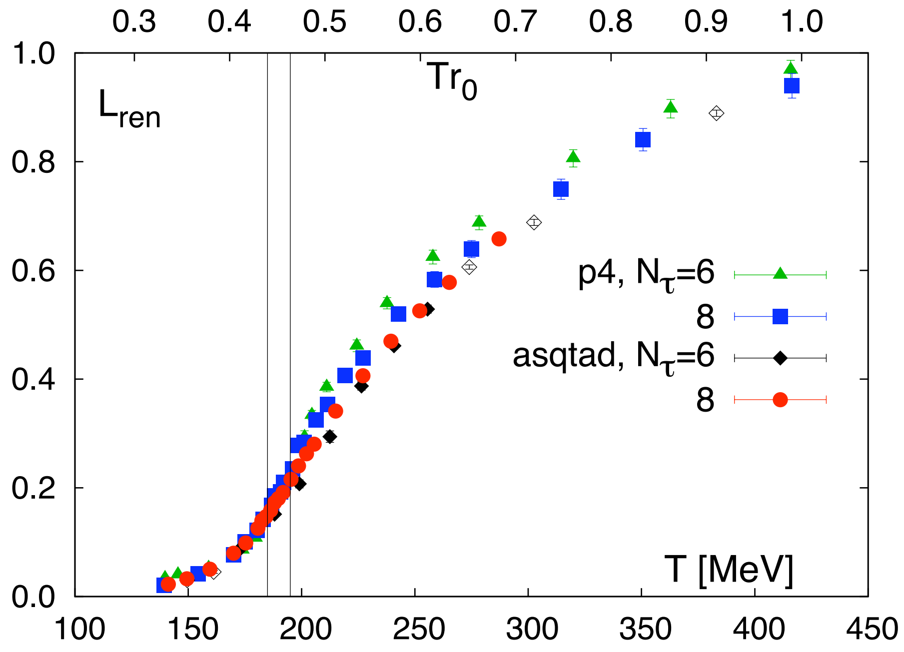
MC, *et. al.*, arXiv: 0911.2215

- Results for p4 action for N_t=8 with m_{ud} = 0.05 m_s
- Extrapolation from results at m_{ud} = 0.20 m_s and m_{ud} = 0.10 m_s imply expected 5 MeV downward shift of transition with decreased mass.
- Results confirm this expectation for T < 200 MeV, but mass dependence perhaps less drastic for T > 200 MeV.

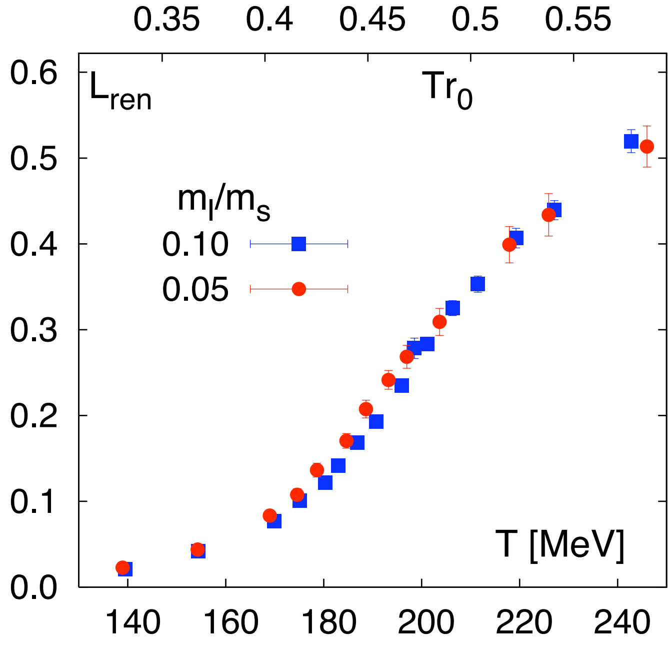


Bazavov, *et. al.,* Phys.Rev.D80:014504,2009

- True order parameter only when quarks decouple (*i.e.* pure gauge theory)
- Polyakov loop related to the free energy of a static quark: L ~ exp(-F/T).
- Needs to be renormalized to remove divergent contributions as a-> 0.
- At high temperature L_{ren} -> 1, reflecting “deconfined” phase.
- Smooth change observed over a large temperature range -> L_{ren} is perhaps a poor probe of singular behavior in theory with light fermions.
- Effect of light quark mass similar to χ_s -> shift to lower temperature.

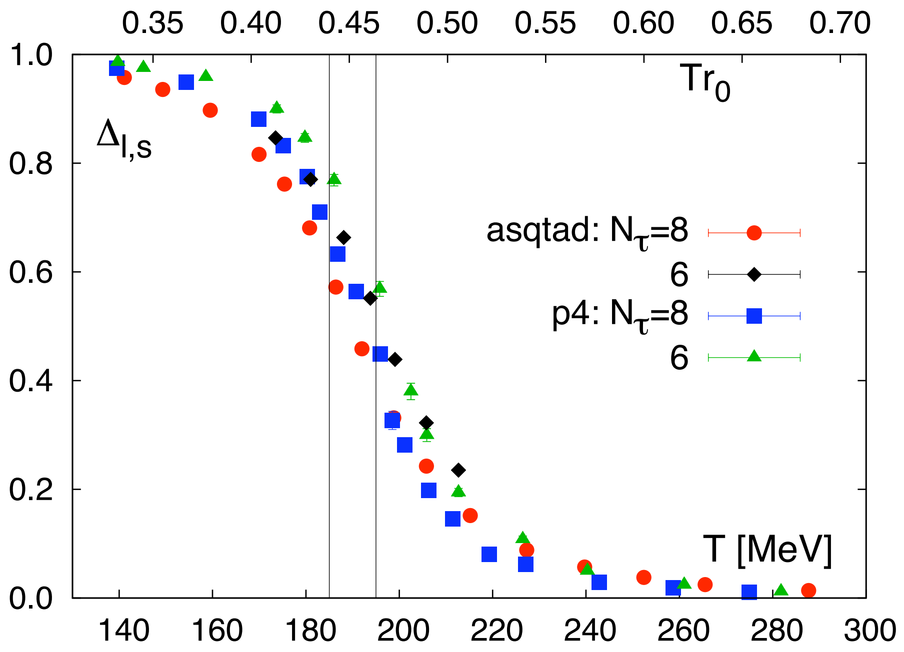


Bazavov, *et. al.,* Phys.Rev.D80:014504,2009



MC, *et. al.*, arXiv: 0911.2215

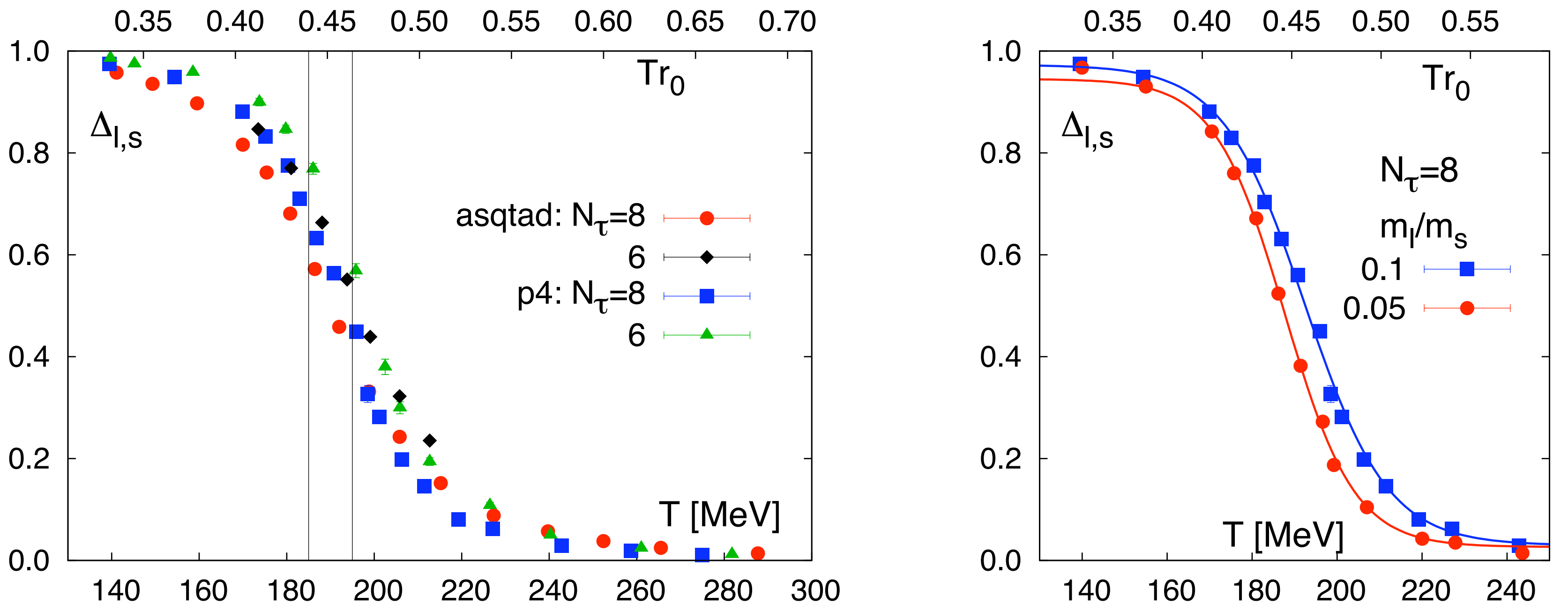
- True order parameter only when quarks decouple (*i.e.* pure gauge theory)
- Polyakov loop related to the free energy of a static quark: L ~ exp(-F/T).
- Needs to be renormalized to remove divergent contributions as a-> 0.
- At high temperature L_{ren} -> 1, reflecting “deconfined” phase.
- Smooth change observed over a large temperature range -> L_{ren} is perhaps a poor probe of singular behavior in theory with light fermions.
- Effect of light quark mass similar to χ_s -> shift to lower temperature.



- Order parameter for chiral symmetry restoration. ($\langle \bar{\psi}\psi \rangle = 0$ in confined phase)

$$\Delta_{l,s}(T) = \frac{\langle \bar{\psi}\psi \rangle_l (T) - \frac{m_l}{m_s} \langle \bar{\psi}\psi \rangle_s (T)}{\langle \bar{\psi}\psi \rangle_l (T=0) - \frac{m_l}{m_s} \langle \bar{\psi}\psi \rangle_s (T=0)}$$

- Larger scaling errors in this quantity than deconfinement observables. However, no evidence in large splitting between deconfinement and chiral restoration.
- Lighter quark mass shifts transition temperature lower, in same way as in deconfinement observables.

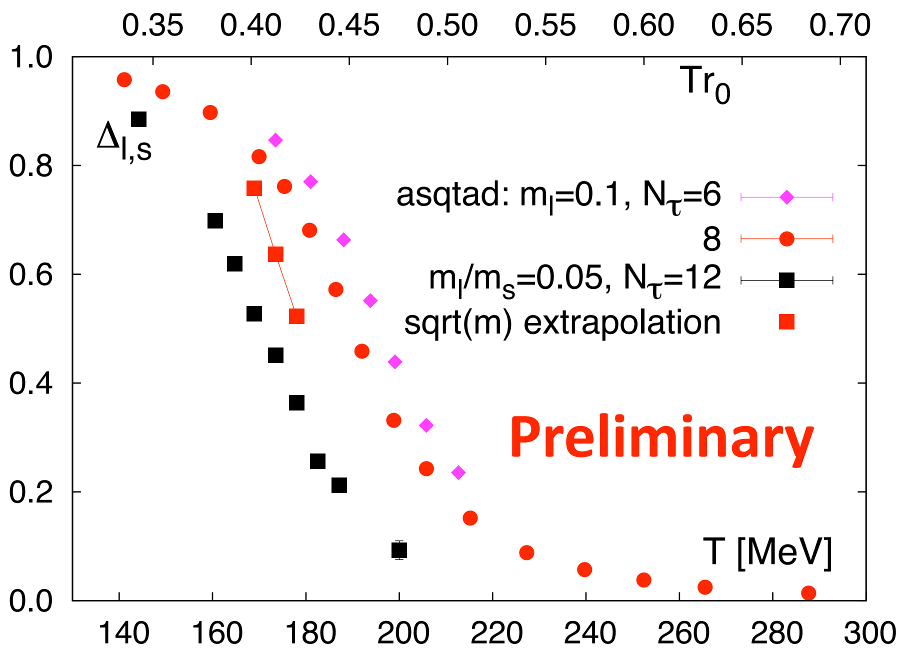


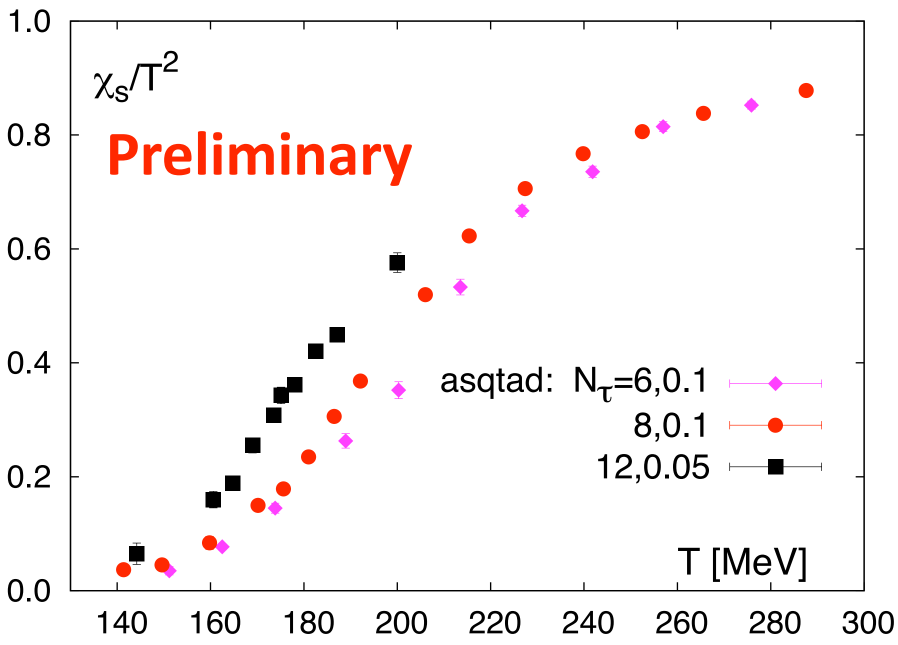
- Order parameter for chiral symmetry restoration. ($\langle\bar{\psi}\psi\rangle = 0$ in confined phase)

$$\Delta_{l,s}(T) = \frac{\left\langle\bar{\psi}\psi\right\rangle_l(T) - \frac{m_l}{m_s}\left\langle\bar{\psi}\psi\right\rangle_s(T)}{\left\langle\bar{\psi}\psi\right\rangle_l(T=0) - \frac{m_l}{m_s}\left\langle\bar{\psi}\psi\right\rangle_s(T=0)}$$

- Larger scaling errors in this quantity than deconfinement observables. However, no evidence in large splitting between deconfinement and chiral restoration.
- Lighter quark mass shifts transition temperature lower, in similar way as in deconfinement observables.

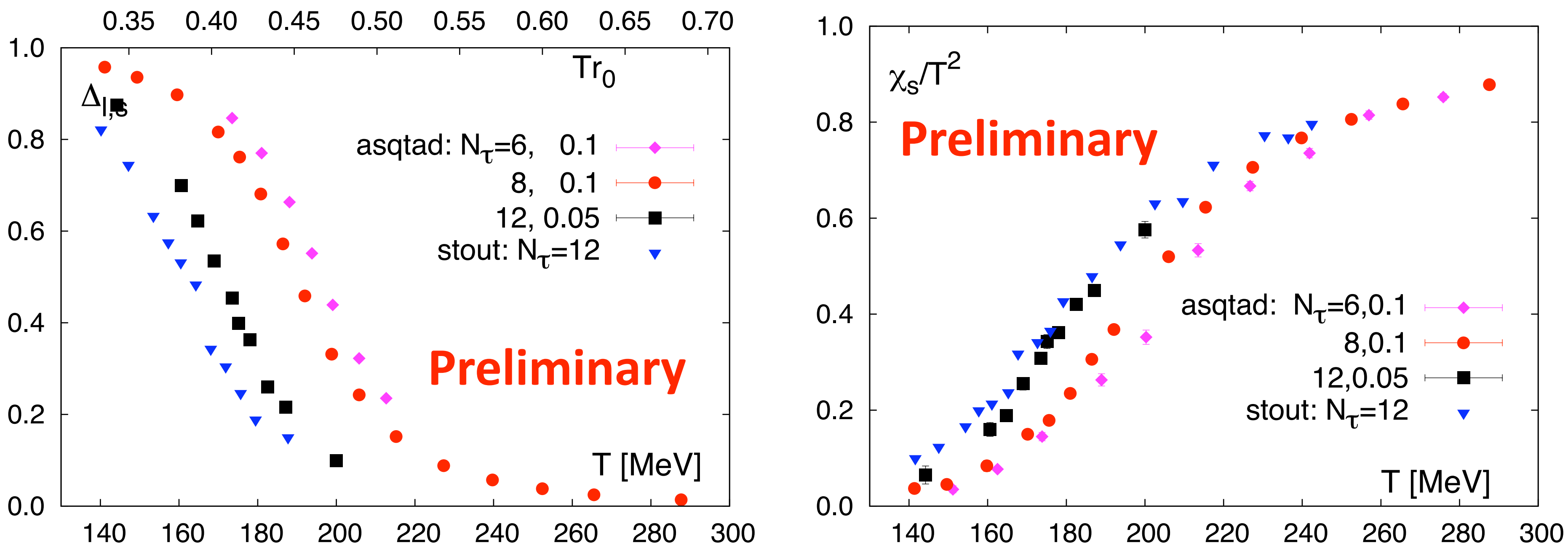
$N_t = 12$



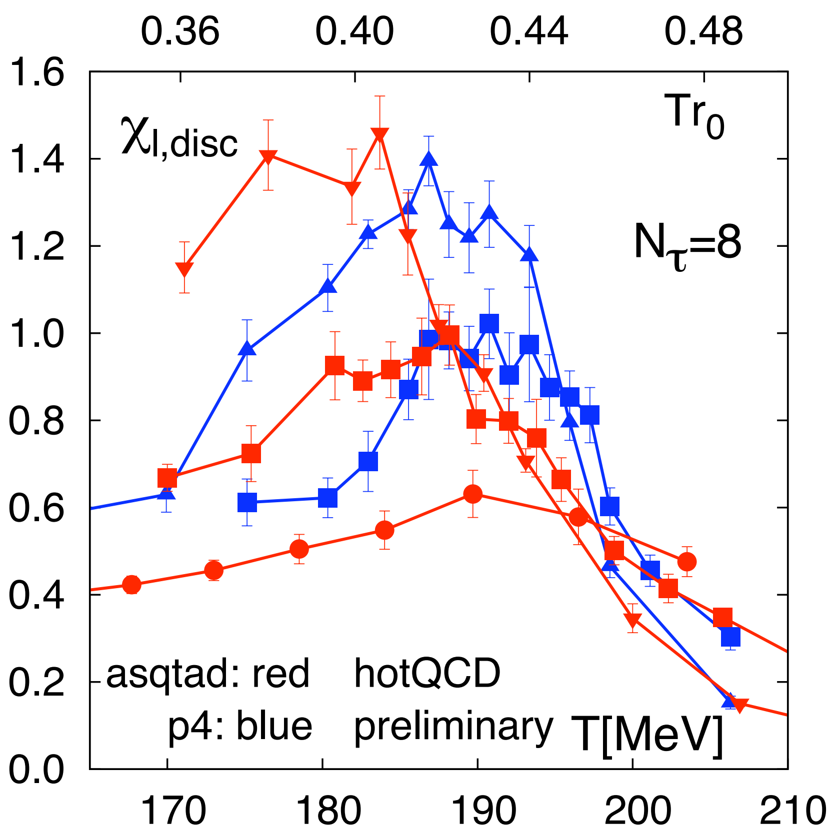


- Preliminary results at N_t = 12 for asqtad action.
- Similar shifts to lower temperature for both chiral and deconfining observables.
- Two things being changed – both quark mass and lattice spacing.

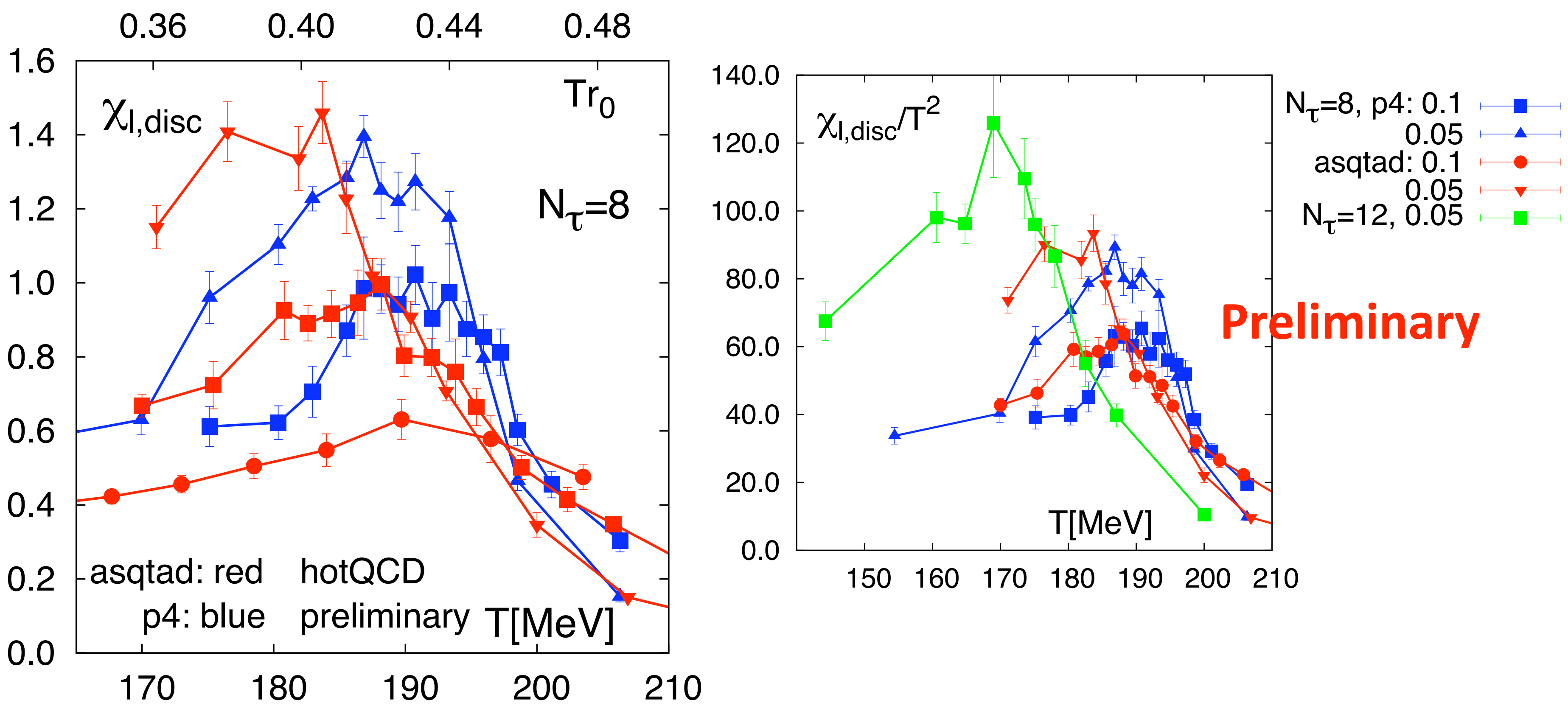
$N_t = 12$



- Comparison with stout N_t = 12 data (scale set using r_0)
- New data shifts χ_s so that it largely agrees with N_t = 12 stout.
- Still discrepancy with stout chiral condensate.
- New data T_c = 170 MeV or less in continuum with physical quark mass.
- However, still no appreciable splitting between deconfinement and chiral.



- Peak in chiral susceptibility can be used to locate T_c.
- O(N) scaling at light quark mass imply asymmetry in chiral susceptibility.
- For T < T_c, there is sqrt(m_q) divergence that pollutes signal for T_c.
- Difficult to pin down T_c for this reason.
- See *e.g.* F. Karsch arXiv:0810.3078



N_t = 12 data shifts curve leftwards, consistent with the other observables.

Conclusion

- Energy density, pressure, entropy density, speed of sound calculated. Pion mass $m_\pi \approx$ 150 MeV at low temperature.
- Small cut-off effects at high temperature. Larger cut-off effects at low temperature -> quark mass effects and flavor symmetry breaking important for comparison with HRG.
- Shift to physical quark mass reduces T_c by about 5 MeV.
- Deconfinement and chiral symmetry observables still give T_c in the same range. Independent of scale setting!
- Preliminary analysis indicates $T_c \sim$ 170 MeV, but not as low as 150 MeV.

References

This talk

- HotQCD: Bazavov, *et. al.,* Phys.Rev.D80:014504,2009 arXiv: 0903.4379
- RBC-Bielefeld: MC, *et. al.*, arXiv: 0911.2215

Other work

- Aoki, *et. al.*, JHEP 0906:088,2009 arXiv: 0903.4155
- Aoki, *et. al.*, Phys.Lett.B643:46-54,2006 hep-lat/0609068
- P. Petreczky and P. Huovinen arXiv:0912.2541
- F. Karsch arXiv:0810.3078