Parton Distributions in Position Space

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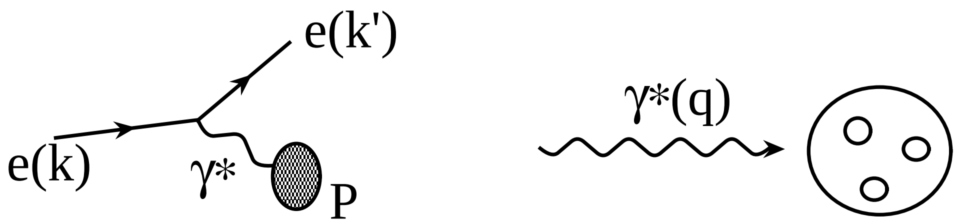
- Generalized parton distributions
- GPDs in Transverse Position Space
- GPDs in Longitudinal Position Space
- Summary

How to Resolve the Proton ?

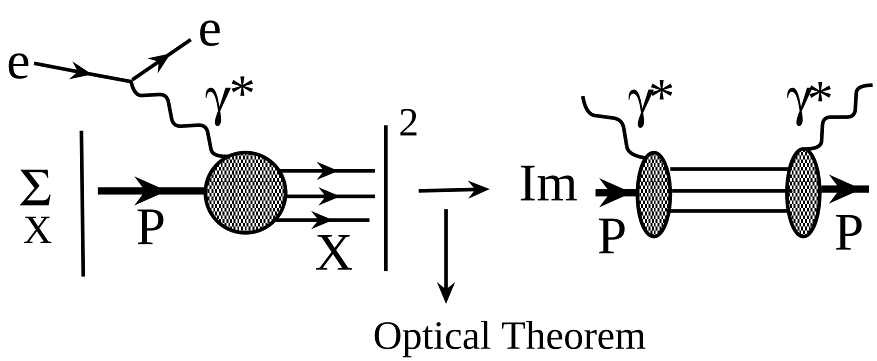
- Proton $\longrightarrow$ made of quarks and gluons
- Proton structure $+$ proton spin $(\frac{1}{2})$
- Experiments using high energy electromagnetic probe acting as a microscope
- Virtual photon of mass (virtuality) $Q^2 = -(k - k')^2$ probes the proton on a distance $r \approx \frac{1}{Q}$.



- Exploring the proton content : (i) Form factors ($ep \longrightarrow ep$); (ii) Parton distributions ($ep \longrightarrow eX$)
- A unifying concept : generalized parton distributions

Deep Inelastic Scattering

- $ep \to eX$ scattering, tool to probe proton structure



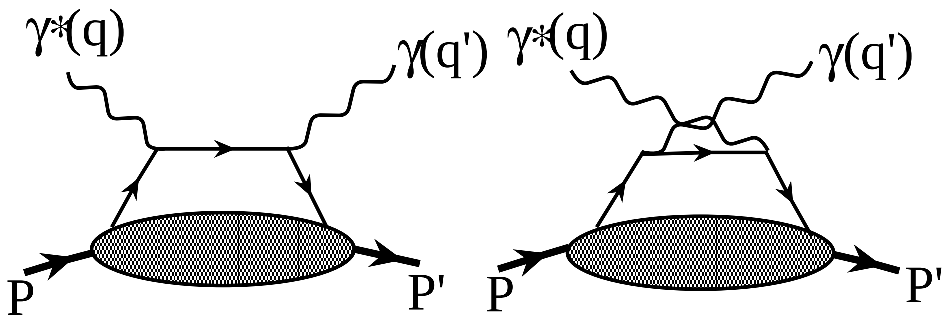
- Bjorken limit : Photon virtuality $Q^2 = -q^2$, and squared hadronic c. m. energy $(P+q)^2$ both large

$x_B = \frac{Q^2}{2P \cdot q}$ fixed

- In this limit, dynamics factorizes into hard partonic subprocess (calculable in perturbation theory) and parton distribution (probability density of finding a parton of specified momentum fraction x in the proton)

- Optical theorem : relates inclusive $\gamma^* p$ cross section to the imaginary part of the forward Compton amplitude $\gamma^* p \to \gamma^* p$.

Deeply Virtual Compton Scattering (DVCS)



- $\gamma^* p \longrightarrow \gamma p$ (exclusive process)
- Momentum of final proton P' different from that of the initial proton
- $Q^2 = -q^2$ large, squared momentum transfer $t = (P' - P)^2$ fixed
- Final photon $\longrightarrow$ real ($q'^2 = 0$)

- Factorization applies : DVCS amplitude $\longrightarrow$ short distance (perturbative part) $*$ 'soft part'

(generalized parton distributions)

DVCS Kinematics and Reference Frame

Consider the process $\gamma^*(q) + p(P) \rightarrow \gamma(q') + p(P')$

Component notation $V^{\pm} = V^0 \pm V^z$ and $V^2 = V^+V^- - (V^{\perp})^2$
Momentum transfer $\Delta = P - P'$, $t = \Delta^2$

Momenta of initial and final proton :

$$P = \left(P^+ \, , \, \vec{0}_{\perp} \, , \, \frac{M^2}{P^+} \right) \, , P' = \left((1-\zeta)P^+ \, , \, -\vec{\Delta}_{\perp} \, , \, \frac{M^2 + \vec{\Delta}_{\perp}^2}{(1-\zeta)P^+} \right)$$

$$t \;=\; 2P \cdot \Delta \;=\; -\frac{\zeta^2 M^2 + \vec{\Delta}_{\perp}^2}{1-\zeta}$$

$\zeta = \frac{Q^2}{2P \cdot q}$: skewness variable;
For DVCS, $-q^2 = Q^2$ is large compared to the masses and $\mid t \mid$

Choose a frame where the incident space-like photon carries $q^+ = 0$

DVCS amplitude

$$M^{IJ}(\vec{q}_\perp, \vec{\Delta}_\perp, \zeta) \; = \epsilon^I_\mu \, \epsilon^{*J}_\nu \, M^{\mu\nu}(\vec{q}_\perp, \vec{\Delta}_\perp, \zeta) \; = -e_q^2 \, \frac{1}{2\bar{P}^+} \int_{\zeta-1}^{1} \mathrm{d}x$$
$$\times \quad \left\{ t^{IJ}(x,\zeta) \, \bar{U}(P') \left[H(x,\zeta,t) \, \gamma^+ + E(x,\zeta,t) \, \frac{i}{2M} \, \sigma^{+\alpha}(-\Delta_\alpha) \right] U(P) \right\},$$

where $\bar{P} = \frac{1}{2}(P' + P)$,
x is the fraction of the proton momentum carried by the active quark
For circularly polarized initial and final photons

$$t^{\uparrow\uparrow}(x,\zeta) = \quad t^{\downarrow\downarrow}(x,\zeta) = \frac{1}{x - i\epsilon} + \frac{1}{x - \zeta + i\epsilon}$$

Contributions from longitudinally polarized photons are suppressed

$$F_{\lambda,\lambda'} = \int \frac{dy^-}{8\pi} \, e^{ixP^+y^-/2} \, \langle P', \lambda' | \bar{\psi}(0) \, \gamma^+ \, \psi(y) \, | P, \lambda \rangle \Big|_{y^+=0, y_\perp=0}$$
$$= \quad \frac{1}{2\bar{P}^+} \, \bar{U}(P',\lambda') \left[H(x,\zeta,t) \, \gamma^+ + E(x,\zeta,t) \, \frac{i}{2M} \, \sigma^{+\alpha}(-\Delta_\alpha) \right] U(P,\lambda),$$

Generalized Parton Distributions : Properties

- GPDs $H(x,\zeta,t)$, $E(x,\zeta,t)$, $\tilde{H}(x,\zeta,t)$ and $\tilde{E}(x,\zeta,t)$ contribute at leading order in $1/Q$
- In the forward limit, $H(x,0,0) = q(x)$ (unpol. quark distribution) and $\tilde{H}(x,0,0) = \Delta q(x,0,0)$ (polarized quark distribution)
- No simple relation for E and $\tilde{E}$ in the forward limit
- Moments of GPDs give nucleon form factors

$$\int_{-1}^{1} dx H(x,\zeta,t) = F_1(t); \qquad \int_{-1}^{1} dx E(x,\zeta,t) = F_2(t)$$

$$\int_{-1}^{1} dx \tilde{H}(x,\zeta,t) = g_A(t); \qquad \int_{-1}^{1} dx \tilde{E}(x,\zeta,t) = g_P(t)$$

- Second moment [X. Ji (1997)]

$$\int_{-1}^{1} dx x [H_q(x,\zeta,t) + E_q(x,\zeta,t)] = A_q(t) + B_q(t)$$

$J_{q,g} = \frac{1}{2}[A_{q,g}(0) + B_{q,g}(0)]; \qquad J_q + J_g = \frac{1}{2}$

- $J_{q,g} \rightarrow$ Fraction of the nucleon spin carried by quarks (gluons)

Why GPDs/DVCS are Interesting ??

- Initial proton momentum differ from final proton momentum : GPDs no longer represent squared amplitudes (like ordinary pdfs) and thus do not have a probability interpretation.

- Richer in content about proton structure than pdfs

- GPDs give a unified picture of both exclusive and inclusive processes

- Second moment of the GPDs gives the fraction of the nucleon spin carried by the quarks.

- Polarization of the target may change due to the scattering : rich spin structure of the GPDs $\rightarrow$ still unknown orbital angular momentum of the partons (quarks, gluons).

- Momentum transfer has a transverse component; leads to information about the transverse structure of the target : impact parameter dependent pdfs

  M. Burkardt, 2000

- 3D spatial structure of the nucleon in terms of the GPDs

  X. Ji, 2003

- Being measured in experiments worldwide : JLab, DESY(HERA and HERMES), CERN (COMPASS).

DVCS Experiments Worldwide

- Fixed target experiments : $x_b > 0.03, Q^2 < 10 \text{ GeV}^2$

1. COMPASS (CERN) : low and medium x_B

$E = 190$ GeV, $0.03 < x_B < 0.25, Q^2 < 7.2\text{GeV}^2$

2. HERMES (Germany) : medium x_B, higher Q^2

$E = 27$ GeV, $0.04 < x_B < 0.3, Q^2 < 9 \text{ GeV}^2$

3. JLab (USA) : medium to large x_B

$E = 6$ GeV, $0.1 < x_B < 0.5, Q^2 < 6 \text{ GeV}^2$

4. Upgraded JLab : larger x_B, higher Q^2

$E = 12$ GeV, $0.05 < x_B < 0.6, Q^2 < 9 \text{ GeV}^2$

Almost no overlap in kinematical regions covered

- Collider experiments : H1 + ZEUS (Germany)

$x_B < 0.01, Q^2 \;\; 5 - 100 \text{ GeV}^2$

Chiral-odd GPDs

Defined as the non-forward matrix elements of light-like correlations of tensor charge
We use the parametrization

$$P^+ \int \frac{dz^-}{2\pi} \qquad e^{\frac{iP^+z^-}{2}} \langle P', \lambda' \mid \bar{\psi}(\frac{-z^-}{2}) \sigma^{+j} \gamma_5 \psi(\frac{z^-}{2}) \mid P, \lambda \rangle_{z^+=0, z_\perp=0}$$

$$= H_T(x,\zeta,t) \bar{u}(P') \sigma^{+j} \gamma_5 u(P) - \tilde{H}_T(x,\zeta,t) \varepsilon^{+j\alpha\beta} \bar{u}(P') \frac{\Delta_\alpha P_\beta}{M^2} u(P)$$

$$- E_T(x,\xi,t) \varepsilon^{+j\alpha\beta} \bar{u}(P') \frac{\Delta_\alpha \gamma_\beta}{2M} u(P) + \tilde{E}_T(x,\zeta,t) \varepsilon^{+j\alpha\beta} \bar{u}(P') \frac{P_\alpha \gamma_\beta}{M} u(P).$$

M. Burkardt (2005), M. Diehl (2003), M. Diehl, P. Haegler (2005)

Momenta of initial and final protons :

$$P = \left(P^+ \, , \, \vec{0}_\perp \, , \, \frac{M^2}{P^+} \right) \, , P' = \left((1-\zeta) P^+ \, , \, -\vec{\Delta}_\perp \, , \, \frac{M^2 + \vec{\Delta}_\perp^2}{(1-\zeta) P^+} \right)$$

Involves quark helicity flip
In the forward limit $H_T(x,0,0)$ reduces to the transversity distribution (generalized transversity)

Chiral Odd GPDs : How to measure ?

- Less investigated than the chiral even counterparts
- Several proposals to measure H_T : for example in photo or electroproduction of two vector mesons on a nucleon target

$$\gamma^* \, p \longrightarrow \rho^0_L \rho^+_T n$$

Ivanov, Pire, Szymanowski, Teryaev (2002), Enberg, Pire, Szymanowski (2006)

Virtual or real photon produces a longitudinally polarized vector meson ρ^0 via two gluon fusion; this meson is separated by a large rapidity gap from other transversely polarized ρ^+ and the scattered neutron

Scattering amplitude factorizes and involves $H_T(x, \xi, 0)$ at zero momentum transfer as well as the chiral odd light-cone distribution amplitude for the transversely polarized meson

Exclusive process $\gamma^* P \longrightarrow \pi^0 P$: to measure the tensor charge

S. Ahmad, G. Goldstein, S. Liuti (2008)

Why Are They Interesting ?

- $\int dx x \left[H_T(x,0,0) + 2\tilde{H}_T(x,0,0) + E_T(x,0,0) \right]$ related to the transverse angular momentum carried by transversely polarized quarks in an unpolarized target : similar to Ji's sum rule

- If quarks are transversely polarized in an unpolarized proton, their distribution is shifted sideways in the transverse plane due to orbital angular momentum.
$\int dx \left[2\tilde{H}_T(x,0,0) + E_T(x,0,0) \right]$ gives information of this shift and spin-orbit correlation

M. Burkardt (2005)

- Fourier transform w.r.t transverse momentum transfer $\Delta_\perp$: impact parameter space distributions; probability interpretation for $\zeta = 0$. For chiral odd GPDs, distortion in impact parameter space gives information about spin-orbit correlations of quarks

- Certain combinations of chiral-odd GPDs also give the correlation between the transverse quark spin and target spin

Light-front Fock representation of DVCS

- P^- is the light-front Hamiltonian, generates x^+ (light-front time) evolution.
$H_{LC} = P^+ P^- - (P^\perp)^2$

- Proton state satisfies $H_{LC} \mid \psi_p \rangle = M^2 \mid \psi_p \rangle$

$$\left|\psi_p(P^+, \vec{P}_\perp)\right\rangle = \sum_n \prod_{i=1}^{n} \frac{\mathrm{d}x_i \, \mathrm{d}^2\vec{k}_{\perp i}}{\sqrt{x_i}\, 16\pi^3} \, 16\pi^3 \delta\left(1 - \sum_{i=1}^{n} x_i\right) \delta^{(2)}\left(\sum_{i=1}^{n} \vec{k}_{\perp i}\right)$$

$$\times \, \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \left| n;\, x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i \right\rangle .$$

- The target state is expanded in terms of multiparticle light-front wave functions in Fock space; choose light-front gauge $A^+ = 0$

DVCS amplitude is given in terms of overlaps of the light-front wave functions

Diehl, Feldman, Jacob, Kroll (2001);
Brodsky, Diehl, Huang (2001)

Overlap representation

- Diagonal parton number conserving $n \to n$ overlap in the kinematical regime $\zeta < x < 1$ and $\zeta - 1 < x < 0$

Off-diagonal $n + 1 \to n - 1$ overlap for $0 < x < \zeta$ where the parton number is decreased by two.

Overlap representation for chiral-odd GPDs involve helicity flip of the quark

Chakrabarti, Manohar, Mukherjee (2008);

- Consider a dressed electron state instead of a proton

State is expanded in Fock space : $| \, e^- \gamma \rangle$ and $| \, e^- e^- e^+ \rangle$ contribute to $O(\alpha)$

- Two and three particle LFWFs are systematically evaluated in perturbation theory

- Generalized form of QED : mass M to the external electrons, m to the internal electron lines λ to the internal photon lines $\longrightarrow$ composite fermion state with mass M : a fermion and a vector "diquark" constituents

Brodsky, Drell (1980)

Generalized Parton Distributions in Impact Parameter Space

Fourier transform with respect to the transverse momentum transfer $\Delta_\perp$ gives GPDs in impact parameter space

$$\begin{aligned}\mathcal{H}(x,\zeta,b_\perp) &= \frac{1}{(2\pi)^2}\int d^2\Delta_\perp e^{-i\Delta_\perp\cdot b_\perp} H(x,\zeta,t)\\ &= \frac{1}{2\pi}\int \Delta d\Delta J_0(\Delta b) H(x,\zeta,t),\end{aligned}$$

where $\Delta = |\Delta_\perp|$ and $b = |b_\perp|$

Other chiral even and chiral odd GPDS in impact parameter space are defined in the same way

Probability interpretation when $\zeta = 0$: impact parameter dependent parton distributions

Nonzero ζ : GPDs in impact parameter space probe partons at transverse position $\mid b_\perp \mid$ with the initial and final proton shited by an amount of order $\zeta \mid b_\perp \mid$.

M. Burkardt (2000), M. Diehl (2001)

GPDs in Longitudinal Position Space

Connection with Wigner distribution : Belitsky, Ji, Yuan (2003)

DVCS amplitude in longitudinal position space : analogy with diffraction pattern in optics

Brodsky, Chakrabarti, Harindranath, Mukherjee, Vary (2006)

We define a boost invariant impact parameter conjugate to the longitudinal momentum transfer as $\sigma = \frac{1}{2} b^- P^+$

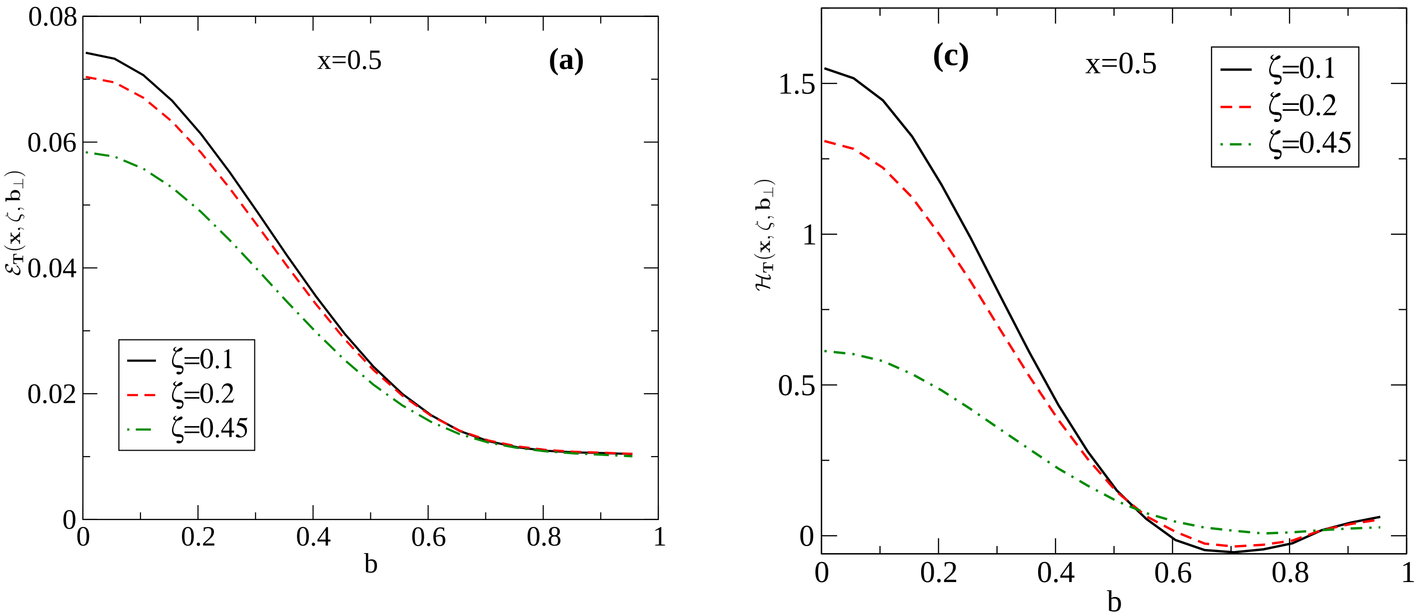
$$\begin{aligned} \mathcal{H}(x,\sigma,t) &= \frac{1}{2\pi}\int_0^{\zeta_f} d\zeta e^{i\frac{1}{2}P^+\zeta b^-} H(x,\zeta,t) \\ &= \frac{1}{2\pi}\int_0^{\zeta_f} d\zeta e^{i\sigma\zeta} H(x,\zeta,t). \end{aligned}$$

Upper limit is the maximum ζ value allowed for fixed $-t$

To get the complete picture both $x > \zeta$ and $x < \zeta$ contributions will have to be considered : chiral even calculated in the reference above

Similarly for other chiral even and chiral odd GPDS

Chiral Odd GPDs in Position Space

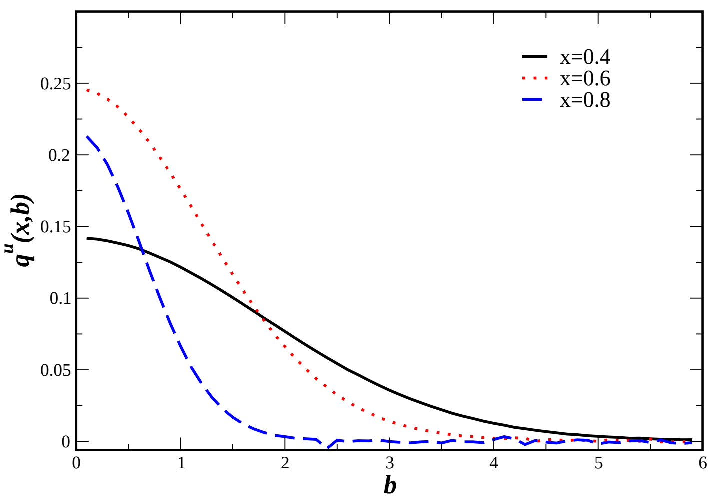


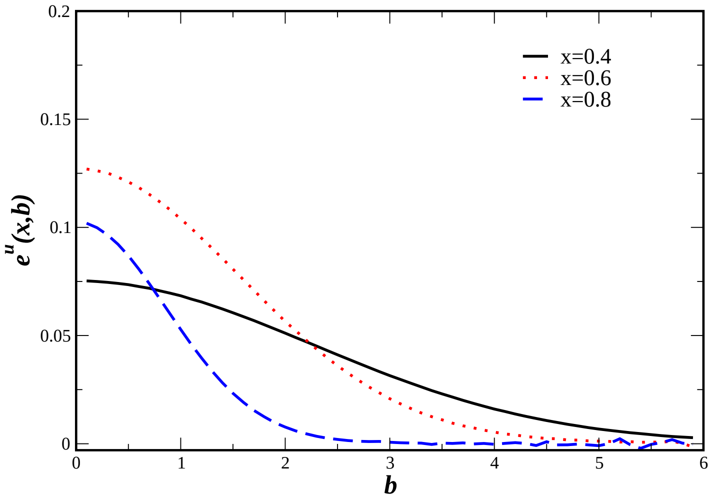
- Instead of a proton, we take an electron at one loop; only diagonal overlap for $x > \zeta$
- E_T for the proton in impact parameter space gives the distortion in the distribution of transversely polarized quarks in an unpolarized proton

- E_T in this model impact parameter space is related to the spin-orbit correlation in the two-particle LFWF.

Chakrabarti, Manohar, Mukherjee (2008);

GPDs for the nucleon in position space





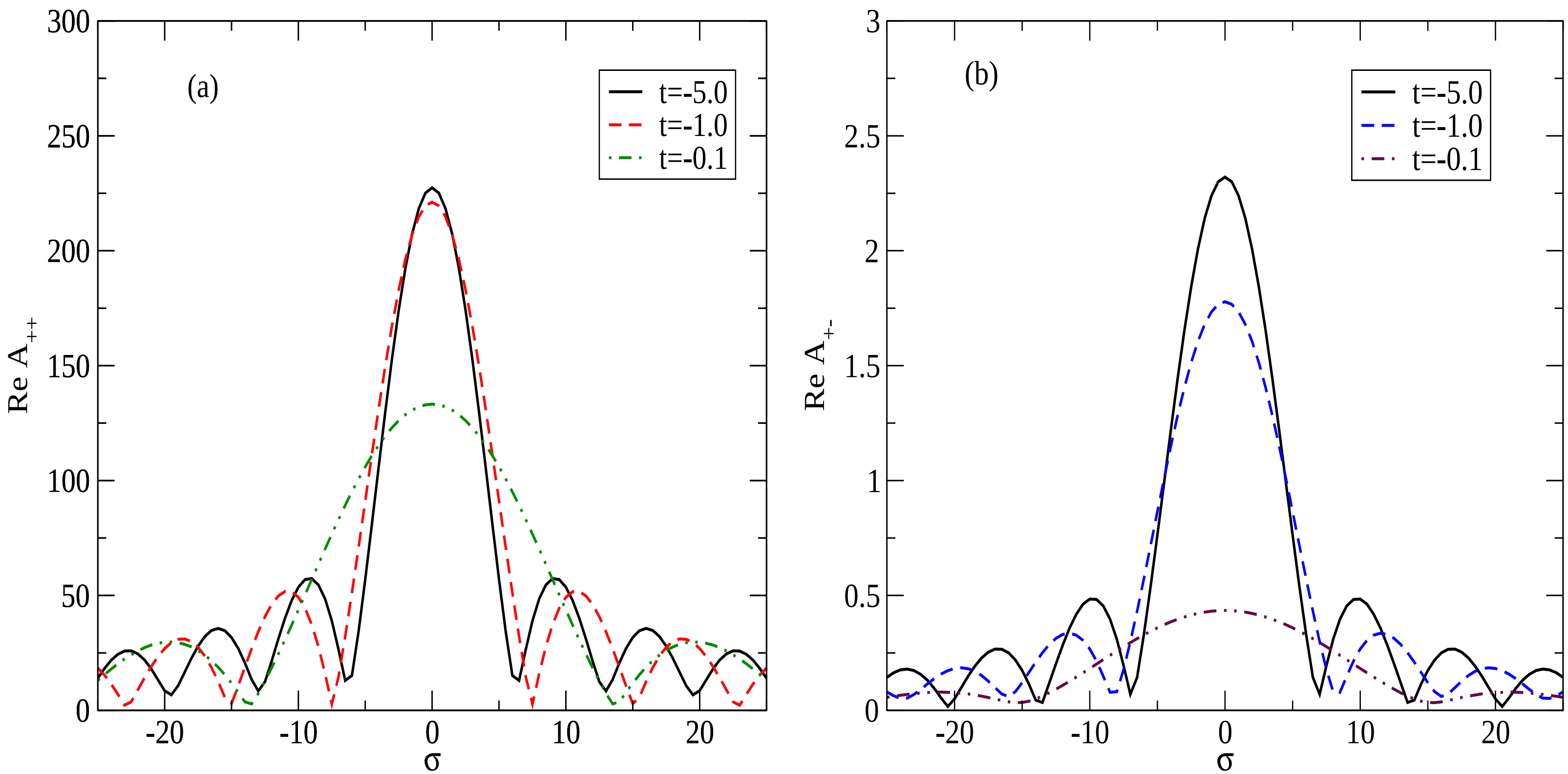
Chakrabarti, Manohar, Mukherjee (2009);

- Used parametrization of

Ahmad, Honkanen, Liuti, Taneja (2007)

- At zero ζ, parametrization obtained by simultaneously fitting the experimental data on nucleon form factor and DIS structure functions
- Spectator model with Regge-type term at input scale 0.09 GeV^2
- For a transverse polarized state, parton distribution is distorted in the transverse position space : related to the FT of E : orbital angular momentum of quarks

Real Part of the DVCS Amplitude in longitudinal position space for an electron at one loop



- (a) When the electron helicity is not flipped, (b) helicity is flipped

$\sigma = \frac{1}{2} P^+ y^-$ is the (boost invariant) longitudinal distance on the light cone; Fourier conjugate of ζ

- Both 2-2 and 3-1 contributions are taken into account : GPDs continuous at $x = \zeta$

- $M = 0.51$ MeV, $m = 0.5$ MeV, $\lambda = 0.02$ MeV, t is in MeV^2

Brodsky,Chakrabarti,Harindranath,AM,Vary; Phys. Rev. D75, 014003 (2007).

Optics Analog

- Analogy with optics :

(i) Finite range of ζ integration act as a slit of finite width and provides a necessary condition for the occurrence of diffraction pattern in the Fourier transform of the DVCS amplitude.
ii) In analogy with optical diffraction, where the positions of the first minima are inversely proportional to the slit width, here we expect their positions to be inversely proportional to ζ_{max}. Since ζ_{max} increases with -t, the position of the first minimum moves to a smaller value of σ.
(iii) For fixed $-t$, higher minima appear at positions which are integral multiples of the lowest minimum : in analogy with diffraction in optics.

- Scattering photons in DVCS provides the complete Lorentz-invariant light front coordinate space structure of a hadron.

Brodsky,Chakrabarti,Harindranath,AM,Vary; Phys.Rev.D75:014003,2007.

Summary

• Discussed chiral even and chiral odd GPDs : their role in understanding the spin structure of the nucleon

• Fourier transform of GPDs : distribution of partons in transverse positon space; interesting interpretation, gives information on the correlation between the intrinsic spin and orbital angular momentum

•Used a a self consistent field theory inspired relativistic two-body model, namely for the quantum fluctuation of an electron at one loop in QED

• Diffraction pattern in longitudinal position space: optics analog