Generalized hadronic susceptibilities and critical behavior of QCD at zero and nonzero density

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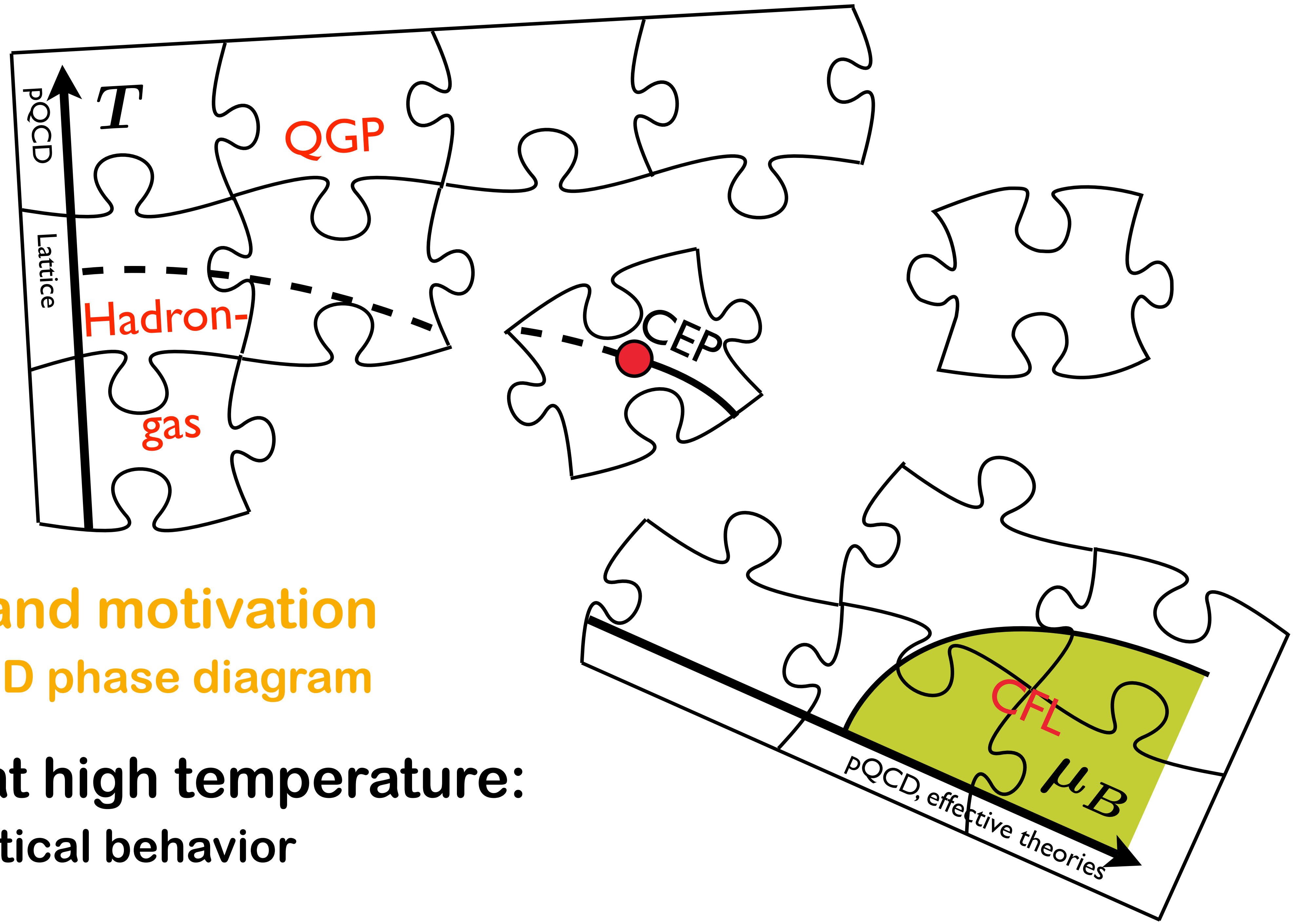


Overview:

★ **Introduction and motivation**
the expected QCD phase diagram

★ **Lattice QCD at high temperature:**
analyzing the critical behavior

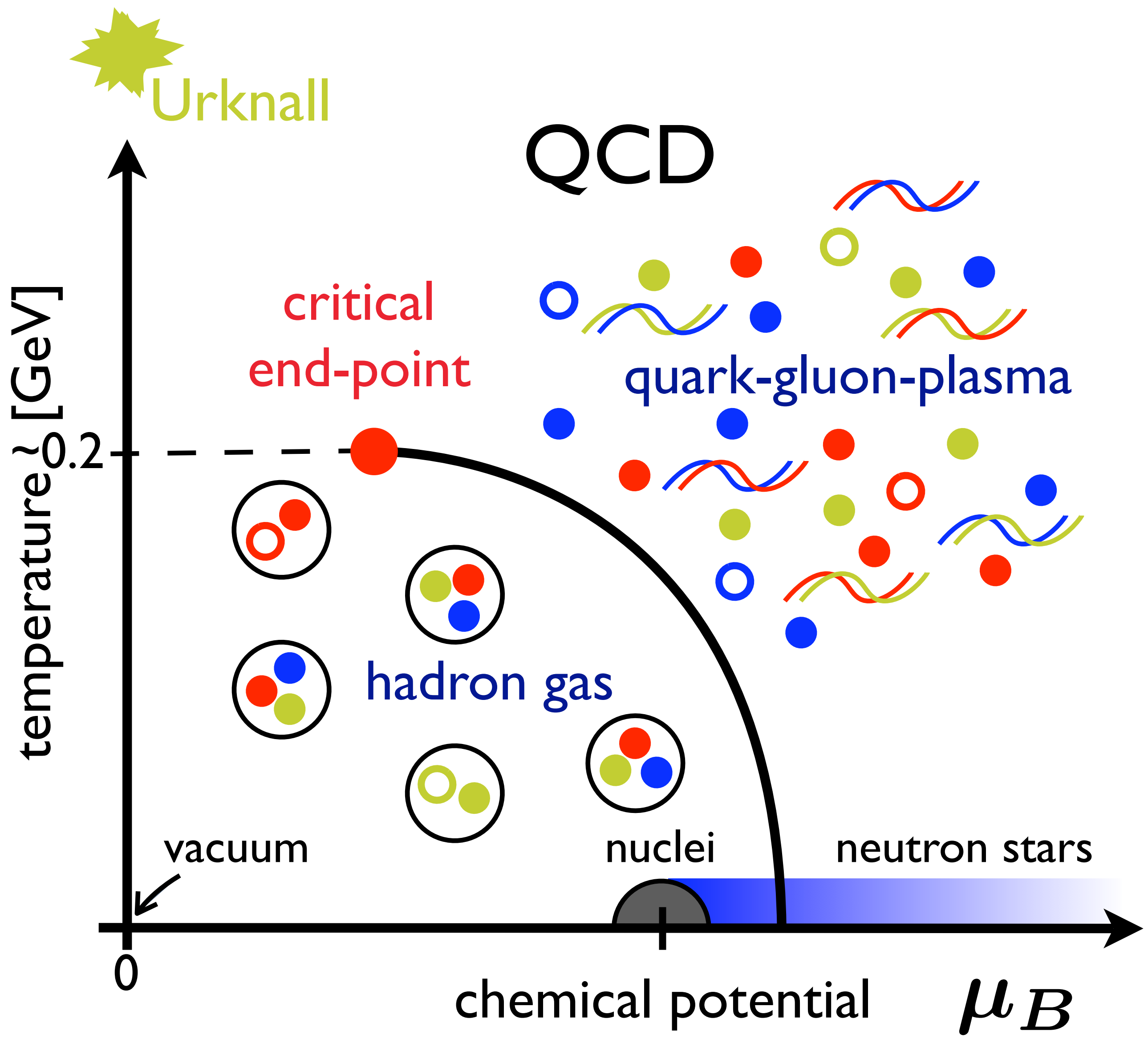
★ **Lattice QCD at high temperature <u>and nonzero density</u>**
Hadronic fluctuations and the critical point

★ **Summary**

The phase diagram

Key questions

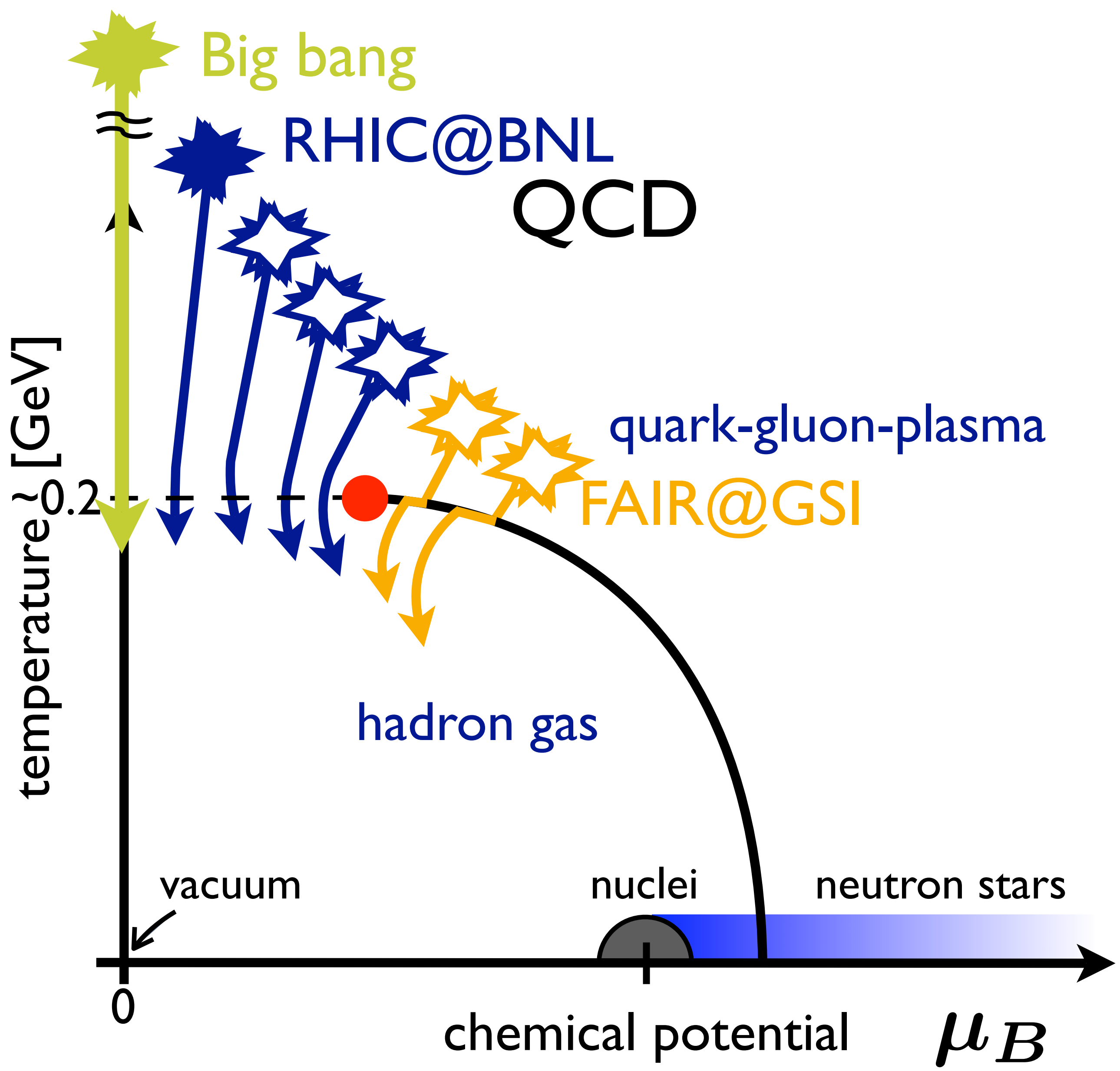
- What are the phases of strongly interacting matter and what role do they play in the cosmos ?
- What does QCD predict for the properties of strongly interacting matter ?
- What governs the transition from Quark and Gluons into Hadrons ?



The QCD phase diagram

Key questions

- What are the phases of strongly interacting matter and what role do they play in the cosmos ?
- What does QCD predict for the properties of strongly interacting matter ?
- What governs the transition from Quark and Gluons into Hadrons ?



Places to find QGP ?

- In the early universe
- In the laboratory: RHIC, LHC, FAIR
- In the cores of neutron stars ?

Analyze critical behavior close to the critical end-point!

The little toolbox for puzzle fans

hadron resonance gas

$$\ln Z(T,V,\mu_B,\mu_S,\mu_Q) \;=\; \sum_{i\in hadrons} \ln Z_{m_i}(T,V,\mu_B,\mu_S,\mu_Q)$$

$$\sum_{i\in mesons} \ln Z^B_{m_i}(T,V,\mu_S,\mu_Q) + \sum_{i\in baryons} \ln Z^F_{m_i}(T,V,\mu_B,\mu_S,\mu_Q)$$

baryons:

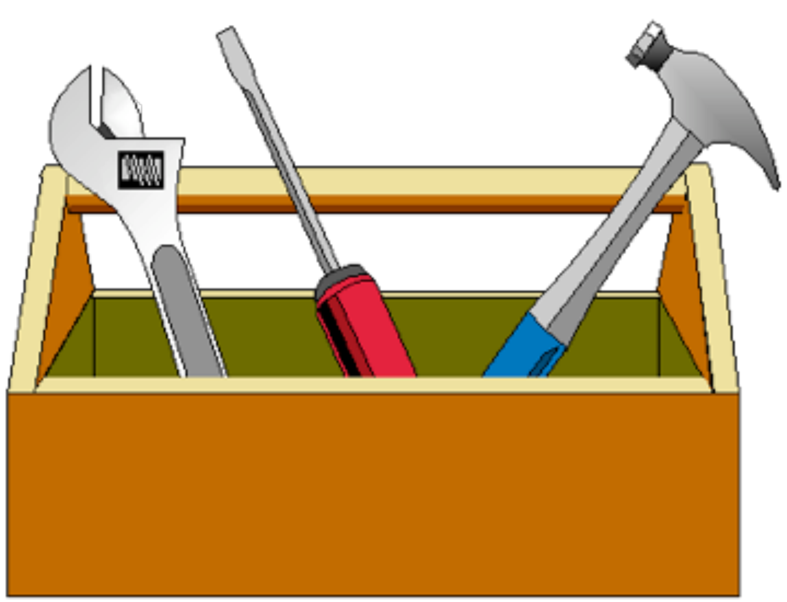
$$\frac{p_i}{T^4} = \frac{d_i}{\pi^2}\left(\frac{m_i}{T}\right)^2 \sum_{l=1}^{\infty} (+1)^{l+1} l^{-2} K_2(lm_i/T) \cosh(lS_i\mu_S/T + lQ_i\mu_Q/T)$$

mesons:

$$\frac{p_i}{T^4} = \frac{d_i}{\pi^2}\left(\frac{m_i}{T}\right)^2 \sum_{l=1}^{\infty} (-1)^{l+1} l^{-2} K_2(lm_i/T) \cosh(lB_i\mu_B/T + lS_i\mu_S/T + lQ_i\mu_Q/T)$$

universal scaling

$$b^d f_s(t,h,\ldots) = f_s(b^{y_t}t, b^{y_h}h,\ldots)$$



perturbation theory ($\mathcal{O}(g^6[\ln(1/g) + \text{const.}])$)

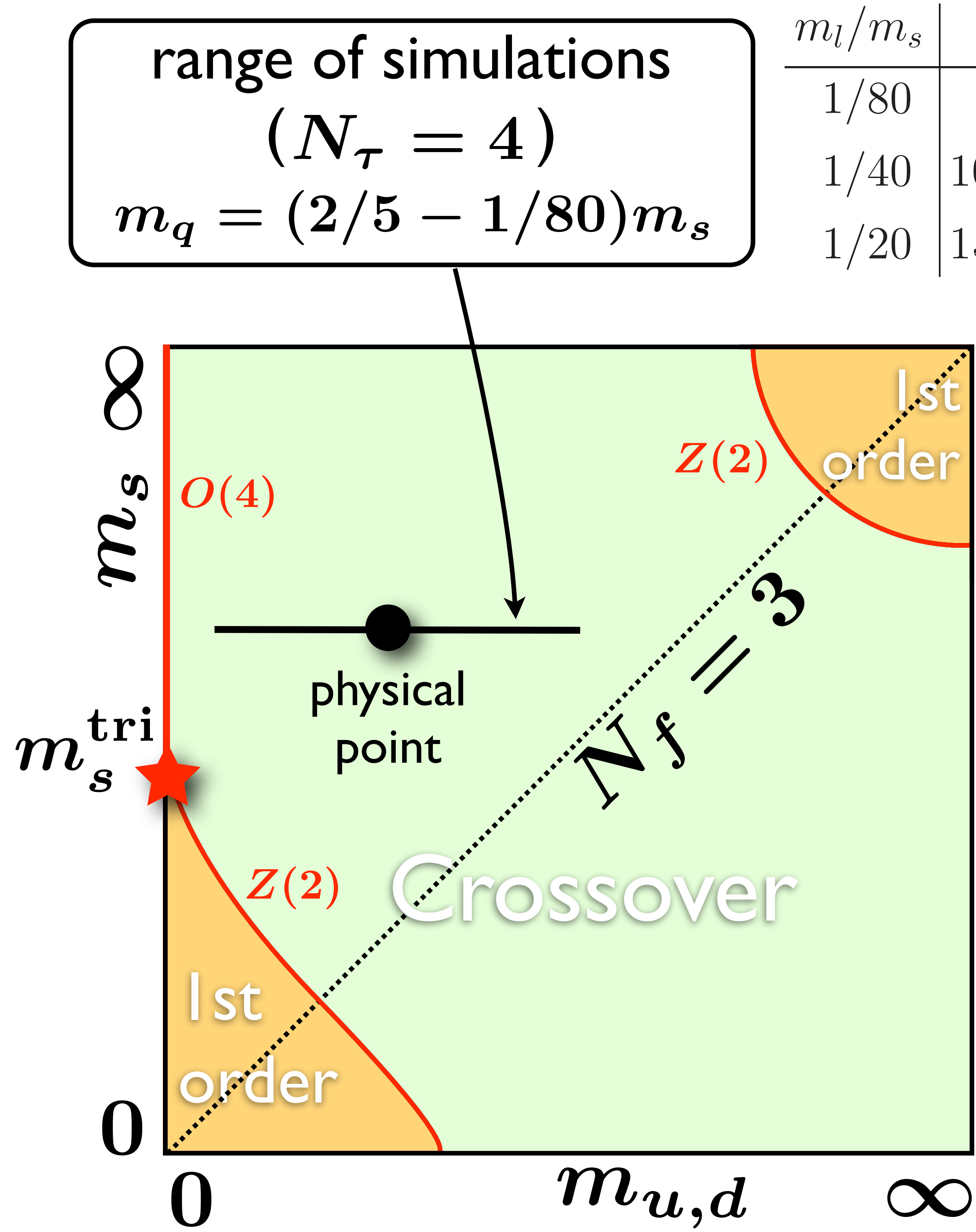
free quark gas ($\mathcal{O}(g^0)$)

$$\frac{p}{T^4} = \frac{8\pi^2}{45} + \sum_{f=u,d,\cdots}\left[\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_f}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_f}{T}\right)^4\right]$$

Simulations with improved staggered fermions (p4fat3)

- chiral symmetry of 2-flavor QCD
  $SU_L(2) \times SU_R(2) \simeq O(4)$
- hence, if expect m_l is large in (2+1)-flavor QCD:
  expect universal behavior as of 3d-$O(4)$ spins in the vicinity of T_c and the chiral limit
- so far no clear evidence from simulations
- staggered fermions preserve a flavor non-diagonal $U(1)$-part of chiral symmetry even at $a > 0$
  $\longrightarrow$ look for $O(2)$-critical behavior

m_l/m_s	m_π
1/80	75 MeV
1/40	105 MeV
1/20	150 MeV

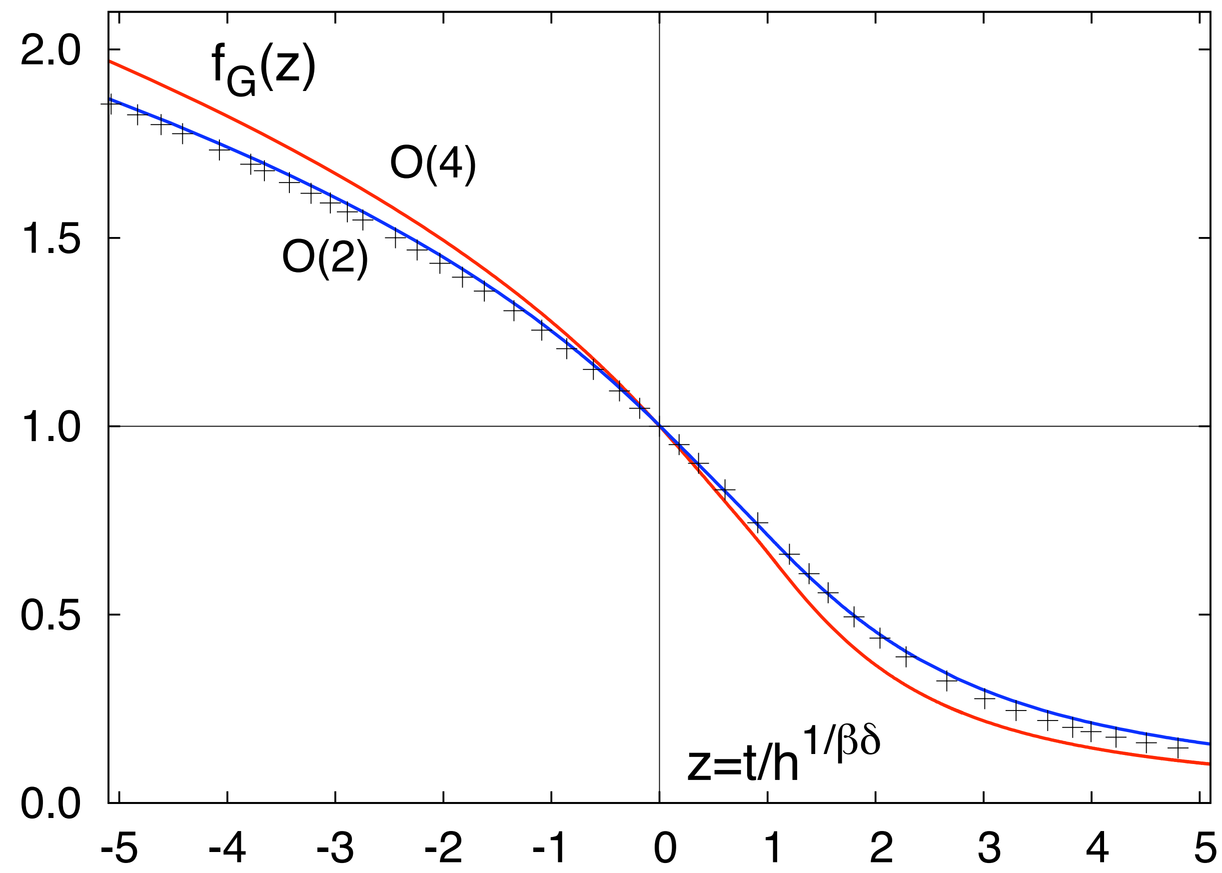


Magnetic EoS in $O(N)$-spin models

- order parameter:

  magnetization $M = h^{1/\delta} f_G(z)$

  universal scaling function



- scaling variable:

  $z = t/h^{1/\beta\delta}$

  where $t = \frac{1}{t_0}\frac{T - T_c}{T_c}$

  (reduced temperature)

  $h = \frac{H}{h_0}$

  (external field)

- scaling function and critical exponents are known to high precision in condensed matter literature [e.g. Engels *et al.*]

- scaling function includes Goldstone effect in the limit of $z \longrightarrow -\infty$

$$z \to -\infty: \quad h \to 0, t < 0 \quad M(t,h) = M(t,0) + c_2(t)\sqrt{h} + \cdots$$

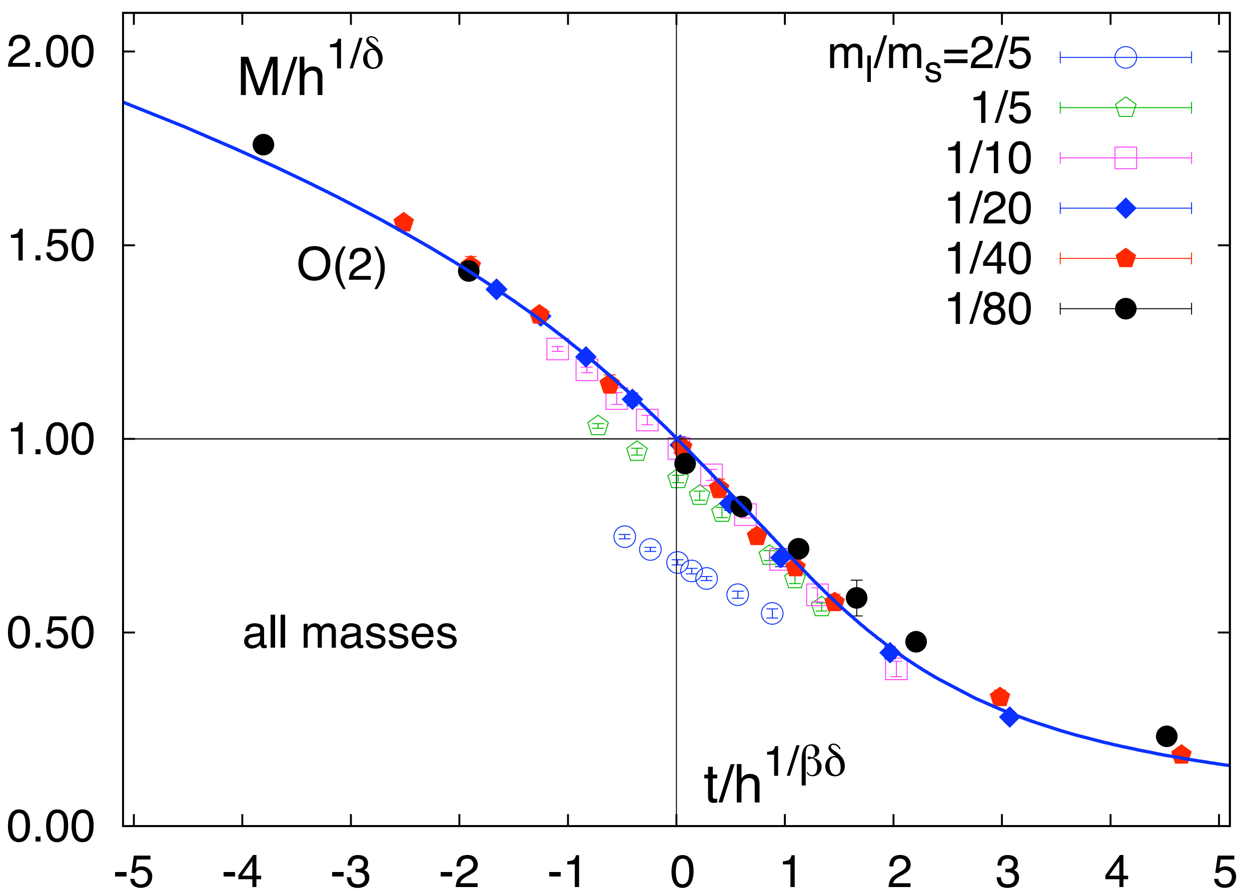
Magnetic EoS in QCD (Nt=4)

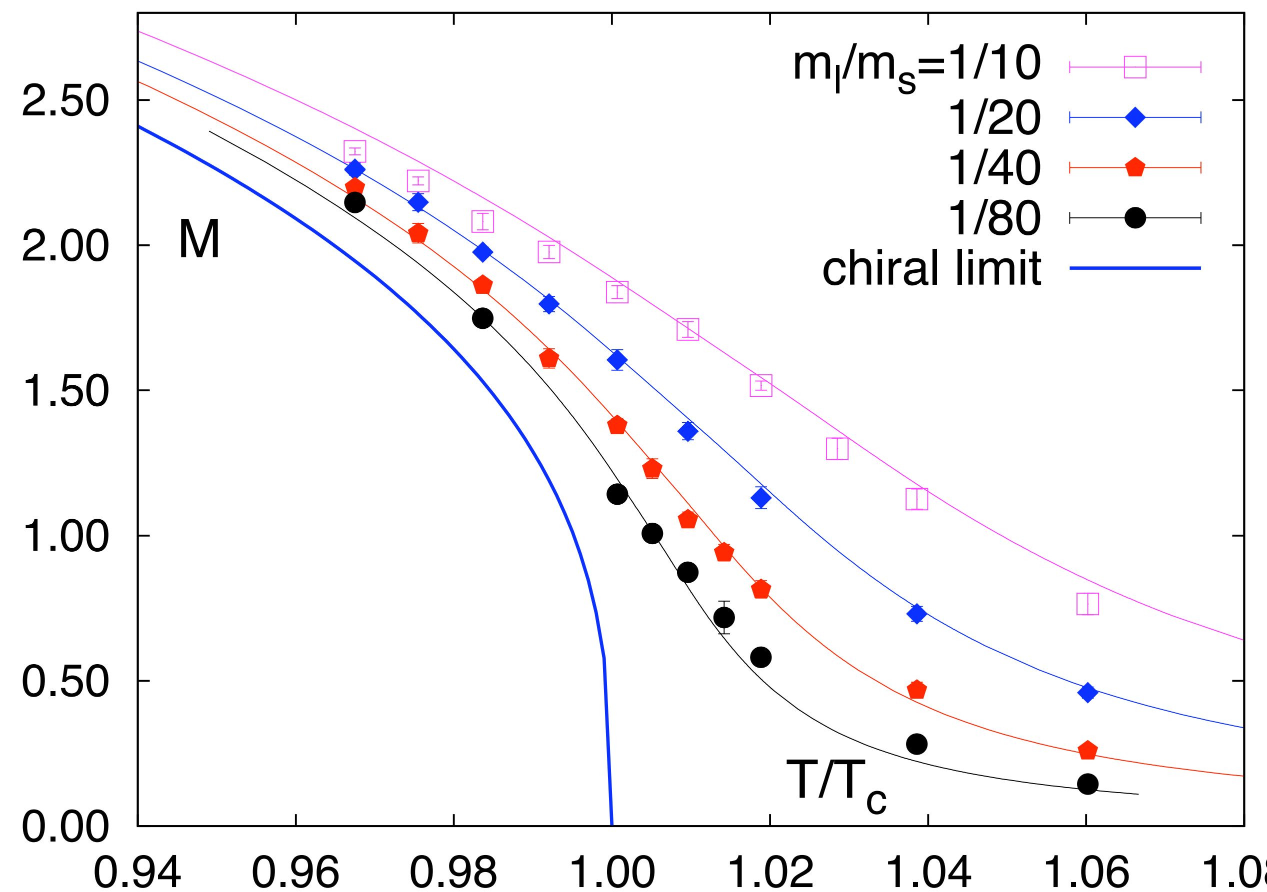
- order parameter: $M = m_s \left(\langle \bar\psi\psi \rangle_l - \frac{m_l}{m_s} \langle \bar\psi\psi \rangle_s \right) N_\tau^4 = h^{1/\delta} f_G(z)$ (chiral condensate)
- scaling variable: $z = t/h^{1/\beta\delta}$

$$t = \frac{1}{t_0} \frac{T - T_c}{T_c} \qquad h = \frac{1}{h_0} \frac{m_l}{m_s}$$

(quark mass)

non-universal constants, determined by fits to the data





→ good agreement with the O(2)-scaling function for $m_l/m_s \leq 1/20$

S. Ejiri *et al.* [RBC-Bielefeld], PRD 80 (2009) 094505.

Universal scaling in QCD (Nt=4)

- order parameter: $M = m_s \left(\langle \bar\psi\psi \rangle_l - \frac{m_l}{m_s} \langle \bar\psi\psi \rangle_s \right) N_\tau^4 = h^{1/\delta} f_G(z)$ (chiral condensate)
- scaling variable: $z = t/h^{1/\beta\delta}$

$$t = \frac{1}{t_0}\frac{T - T_c}{T_c} \qquad h = \frac{1}{h_0}\frac{m_l}{m_s}$$

(quark mass)

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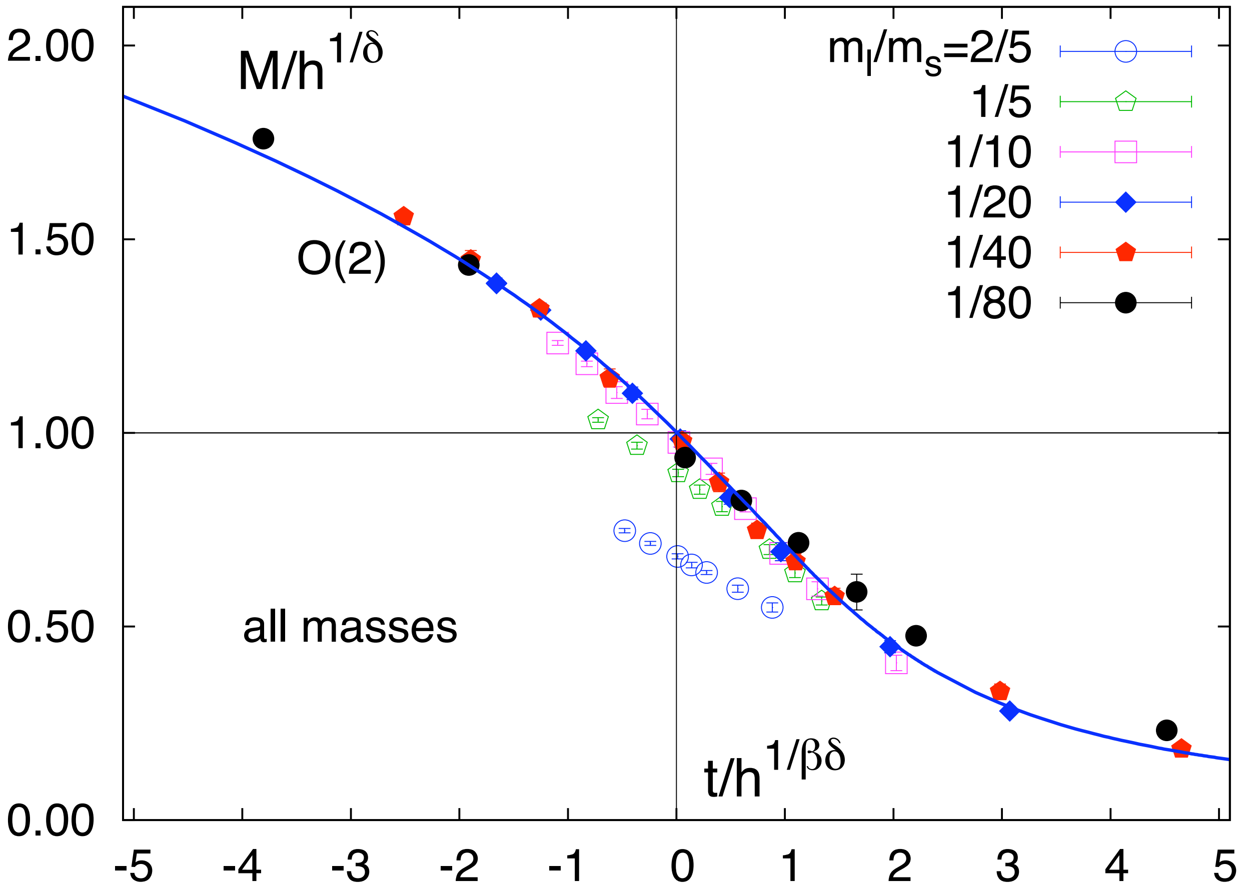


$$z_0 = h_0^{1/\beta\delta}/t_0 \approx 8.0(8)$$

unique combination

$$\frac{T_p(m_l/m_s) - T_c}{T_c} = \frac{z_p}{z_0}(m_l/m_s)^{1/\beta\delta}$$

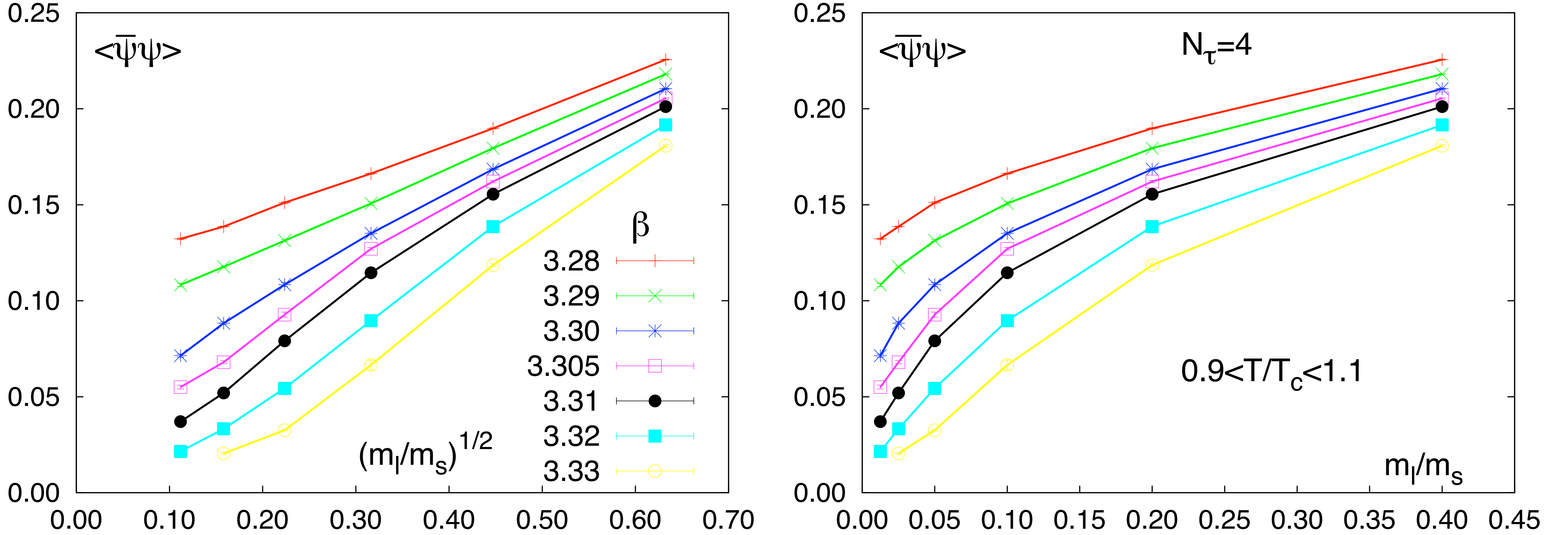
mass dependence of the transition temperature

→ good agreement with the O(2)-scaling function for $m_l/m_s \leq 1/20$

S. Ejiri *et al.* [RBC-Bielefeld], PRD 80 (2009) 094505.

Goldstone effect

$$\langle \bar\psi\psi\rangle = \frac{1}{N_\sigma^3 N_\tau}\frac{n_f}{4}\frac{\partial \ln Z}{\partial m_l a}$$



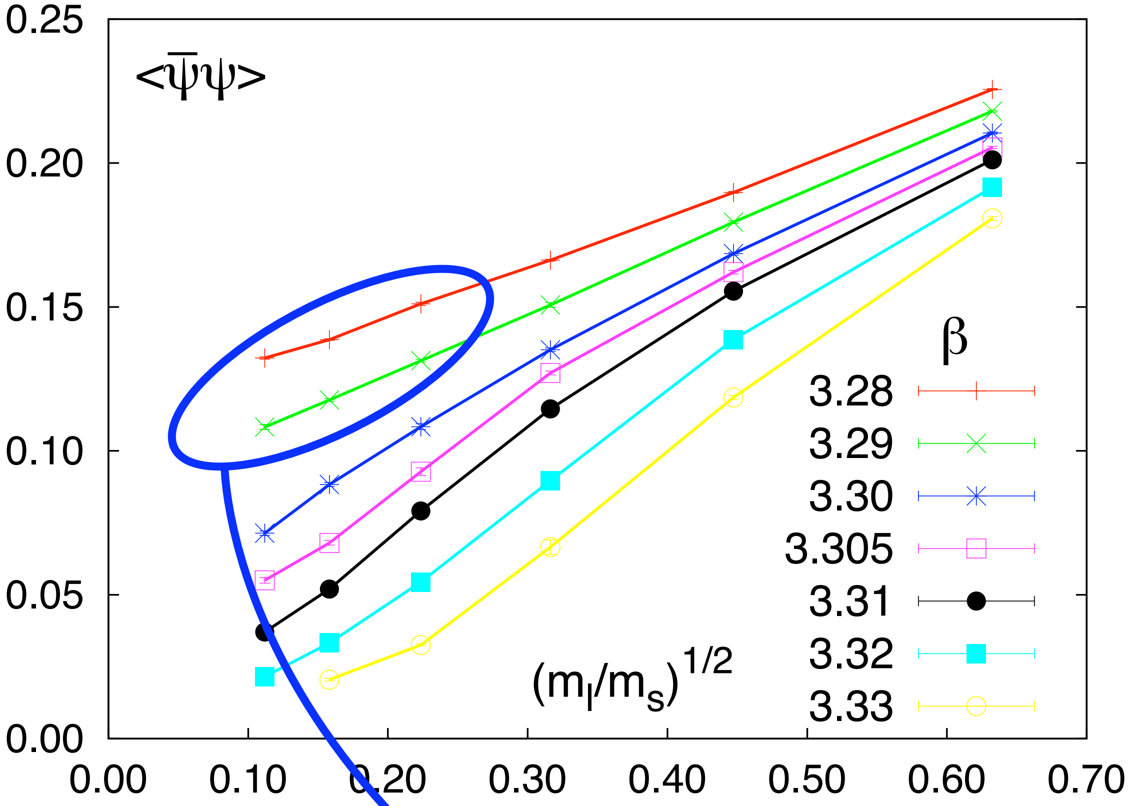
$\beta = 3.28\ T \simeq 188$ MeV, $\beta = 3.30\ T \simeq 196$ MeV

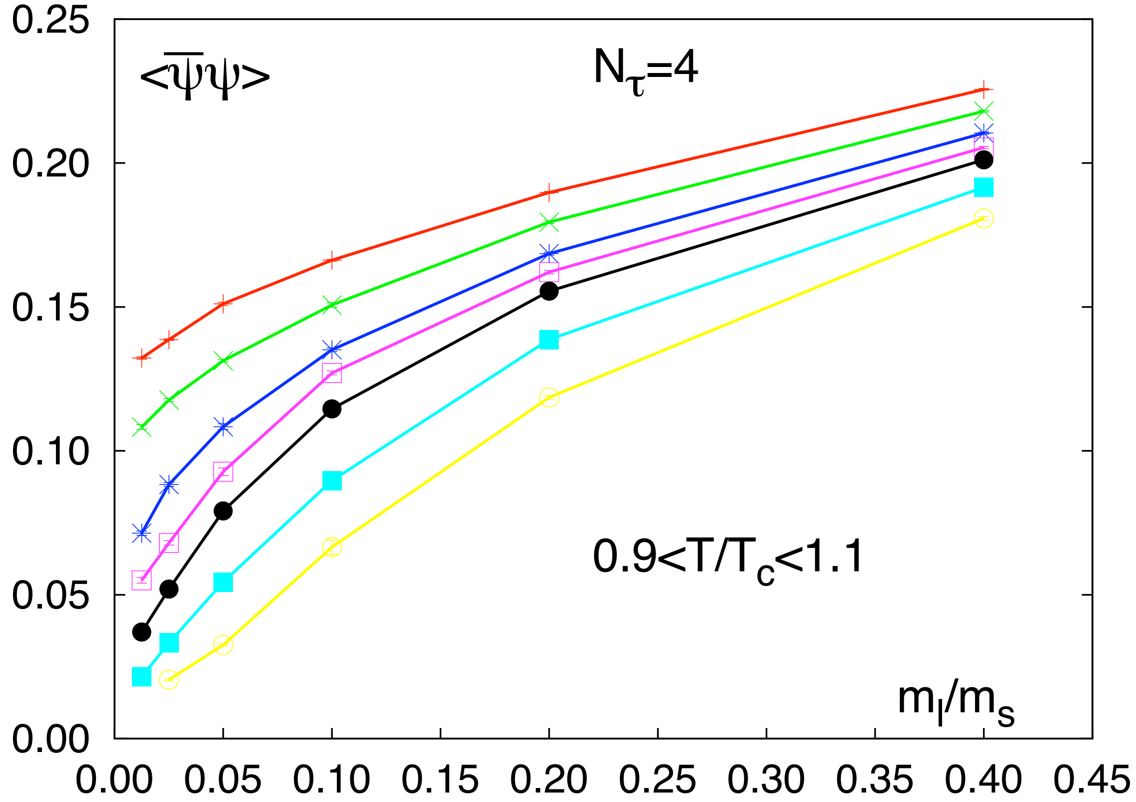
Goldstone mode influences chiral limit for $T < T_c$:

$$\langle \bar\psi\psi\rangle \sim \begin{cases} c(T)\sqrt{m_q} + d(T)m_q + \text{regular} & T < T_c \\ c_\delta m_q^{1/\delta} + d(T_c)m_q + \text{regular} & T = T_c \\ d(T)m_q + \text{regular} & T > T_c \end{cases}$$

Goldstone effect

$$\langle\bar{\psi}\psi\rangle = \frac{1}{N_\sigma^3 N_\tau}\frac{n_f}{4}\frac{\partial \ln Z}{\partial m_l a}$$





clear evidence of Goldstone effect

$\beta = 3.28$ $T \simeq 188$ MeV, $\beta = 3.30$ $T \simeq 196$ MeV

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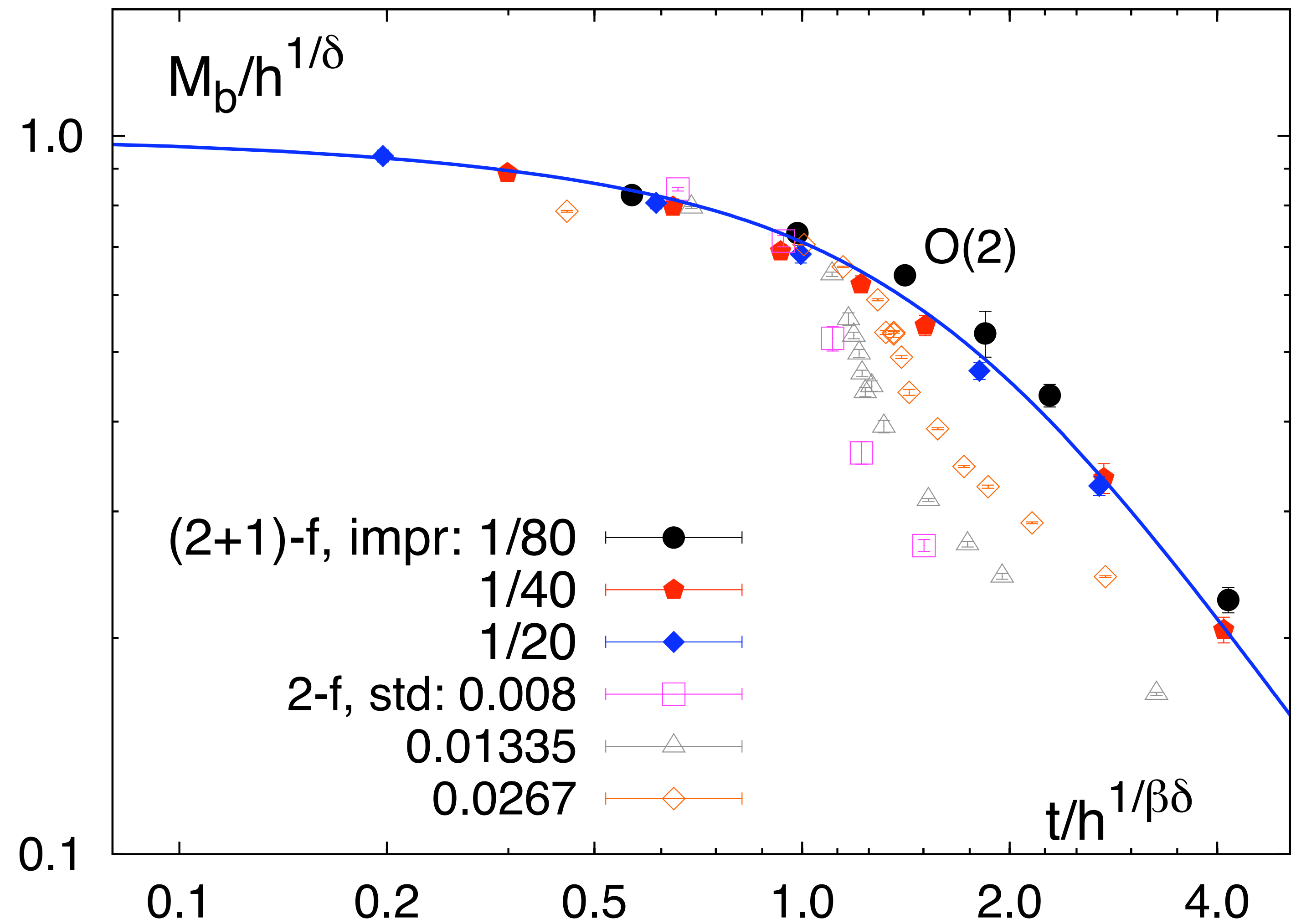
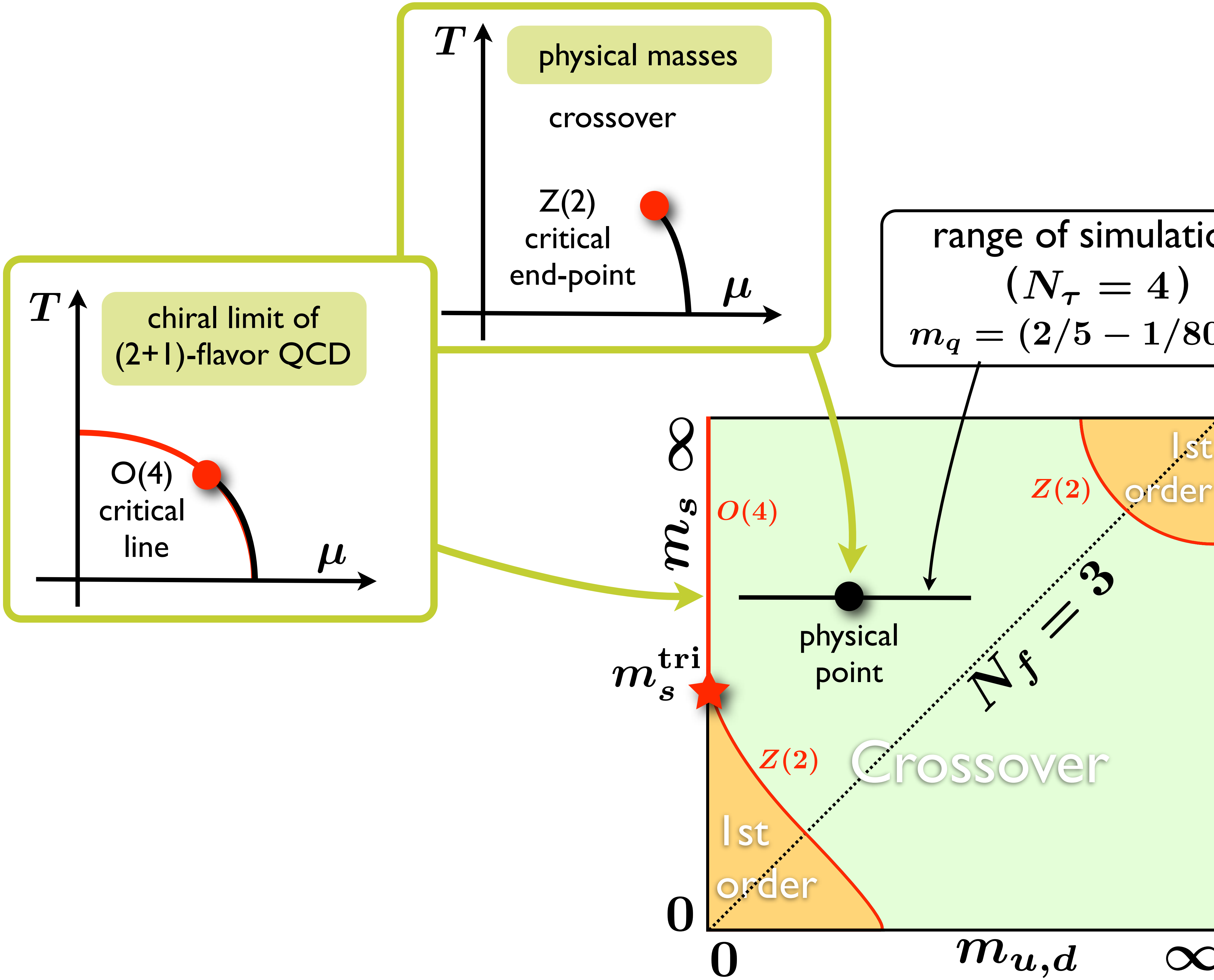
Comparison with previous data

- until now comparison only for $z > 0$
- similar difficulties with fits for critical exponents



includes MILC, Pisa data
[2-flavor std. staggered fermions]

Extention to nonzero chemical potential



Lattice QCD at nonzero density (I)

- direct MC-simulations for $\mu > 0$ not possible

$$Z(V,T,\mu) = \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \exp\{S_F(A,\psi,\bar{\psi}) - \beta S_G(A)\}$$

$$= \int \mathcal{D}A\ \det[M](A,\mu)\exp\{-\beta S_G(A)\}$$

complex for $\mu > 0$

Interpretation as probability is necessary for MC-Integration

$\longrightarrow$ perform a Taylor expansion around $\mu = 0$

Lattice QCD at nonzero density (I)

- Taylor-expansion of the pressure

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu_u,\mu_d,\mu_s) = \sum_{i,j,k} c_{i,j,k}^{u,d,s} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k$$

- calculate Taylor coefficients at fixed temperature

- no sign-problem: all simulations at $\mu = 0$

$$c_{i,j,k}^{u,d,s} \equiv \frac{1}{i!j!k!} \frac{1}{VT^3} \cdot \left. \frac{\partial^i \partial^j \partial^k \ln Z}{\partial(\frac{\mu_u}{T})^i \partial(\frac{\mu_d}{T})^j \partial(\frac{\mu_s}{T})^k} \right|_{\mu_{u,d,s}=0}$$

- expansion coefficients reflect fluctuations of various quantum numbers

generalized susceptibilities

$$2! c_2^X = \chi_2^X = \frac{1}{VT^3} \left(\langle X^2 \rangle - \langle X \rangle^2 \right)$$ quadratic fluctuations

$$4! c_4^X = \chi_4^X = \frac{1}{VT^3} \left(\langle X^4 \rangle - 3 \langle X^2 \rangle^2 \right)$$ quartic fluctuations

$$X = u, d, s, B, Q, S, \cdots$$

Lattice QCD at nonzero density (I)

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$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu_B,\mu_Q,\mu_S) = \sum_{i,j,k} c_{i,j,k}^{B,Q,S} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

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$$X = u, d, s, B, Q, S, \cdots$$

Hadronic fluktuations (I)

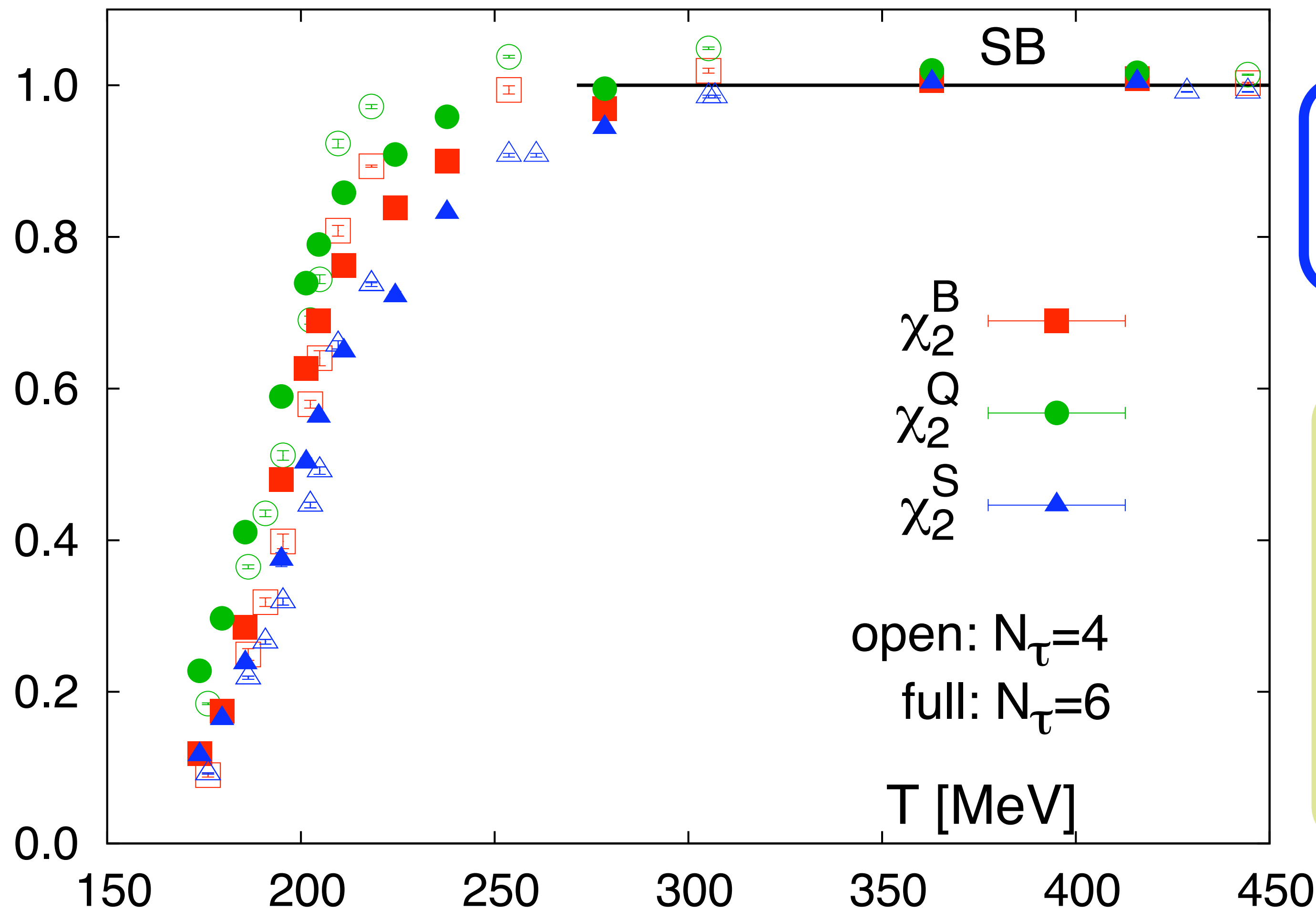
Quadratic fluctuations (c_2)

$$\chi_X^2 = \frac{1}{VT^3}\left(\langle X^2\rangle - \langle X\rangle^2\right)$$

$$X = B, Q, S$$

$T < T_c$

conserved charges are carried by massive hadrons



$T > T_c$

conserved charges carried by light quarks

- lattice effects are small
- agreement with the free quark gas for $T > 1.5T_c$

$T \approx T_c$

temperature dependence dominated by the regular part of the free energy: **similar to energy density**

$$\chi_X^2 \propto |T - T_c|^{1-\alpha} + \text{regular}$$

$$\alpha \approx -0.25$$

Hadronic fluktuations (I)

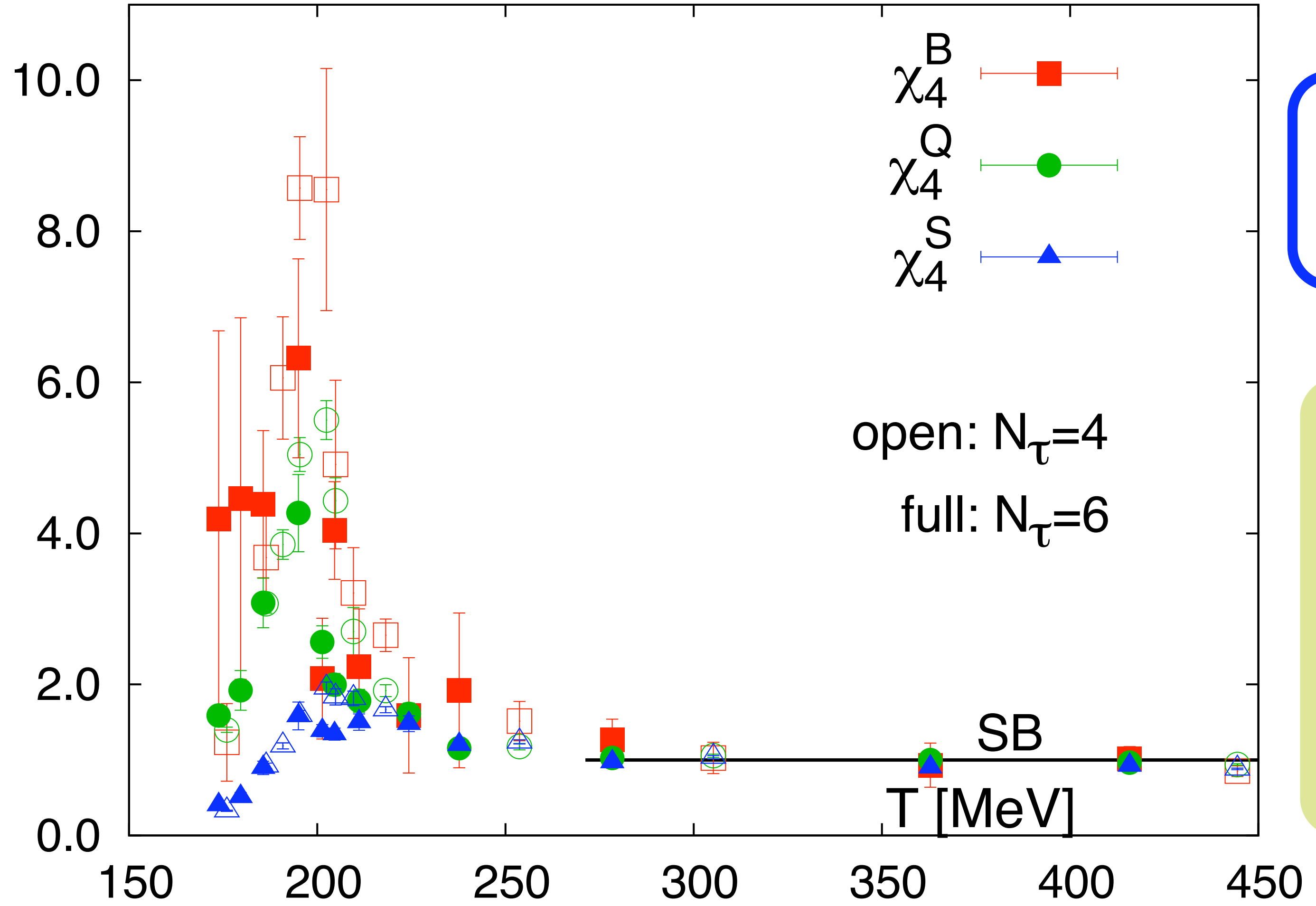
Quartic fluctuations (c_4)

$$\chi_X^4 = \frac{1}{VT^3}\left(\langle X^4\rangle - 3\langle X^2\rangle^2\right)$$

$$X = B, Q, S$$

$T < T_c$

conserved charges are carried by massive hadrons



$T > T_c$

conserved charges carried by light quarks

- lattice effects are small
- agreement with the free quark gas for $T > 1.5T_c$

$T \approx T_c$

critical behavior:
similar to specific heat

$$\chi_X^2 \propto |T - T_c|^{-\alpha} + \text{regular}$$

$$\alpha \approx -0.25$$

Hadronic fluctuations (III)

at $\mu_B > 0$ $(\mu_S = \mu_Q = 0)$

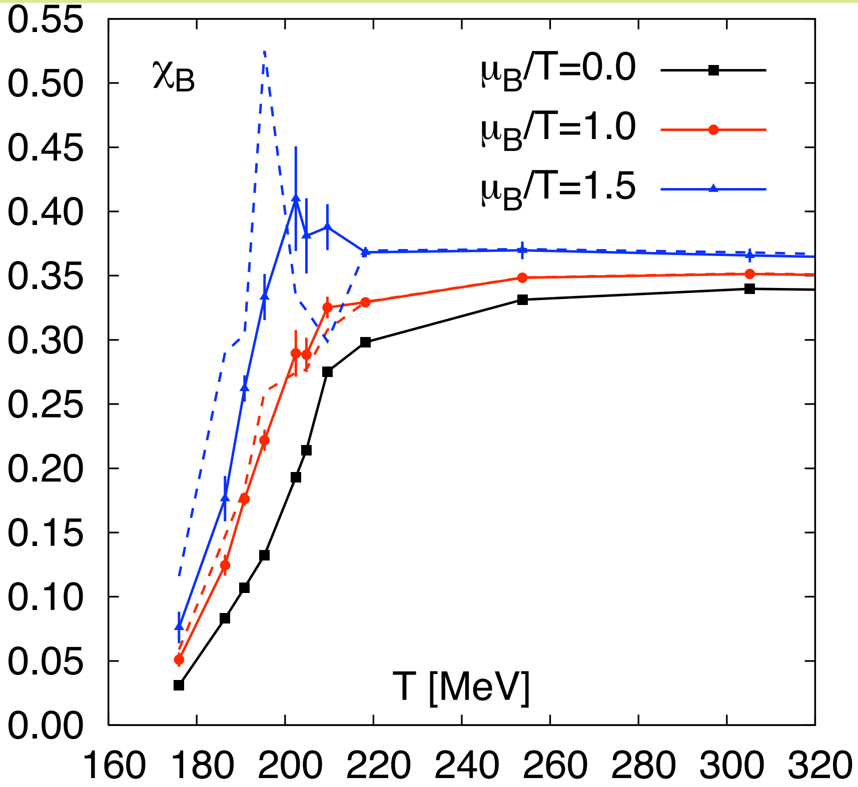
baryon number fluctuations

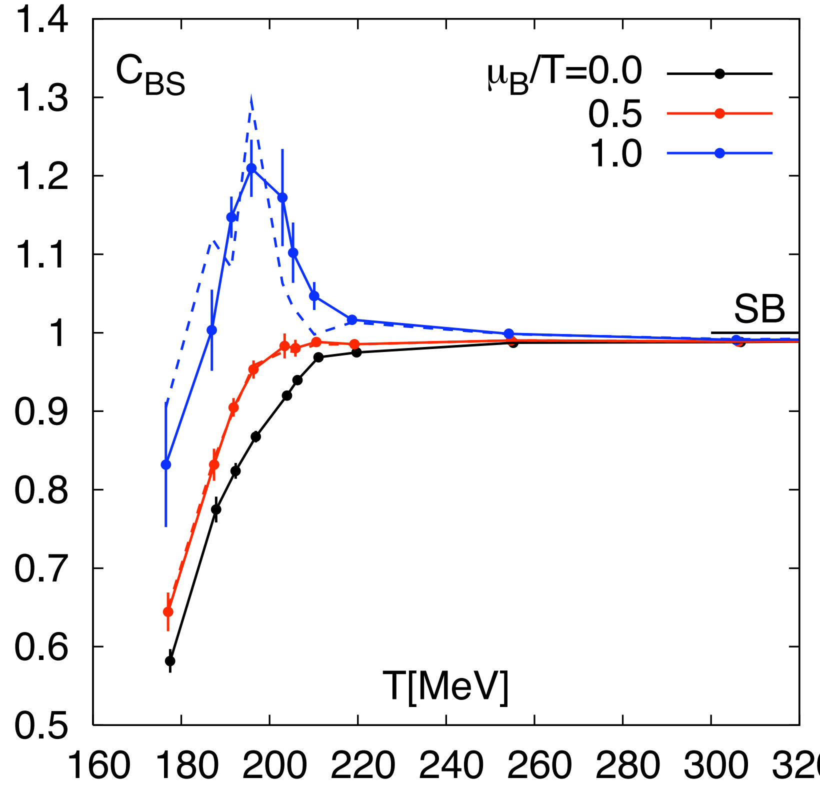
$$\chi_B = 2c_2^B + 12c_4^B \left(\frac{\mu_B}{T}\right)^2 + \cdots$$

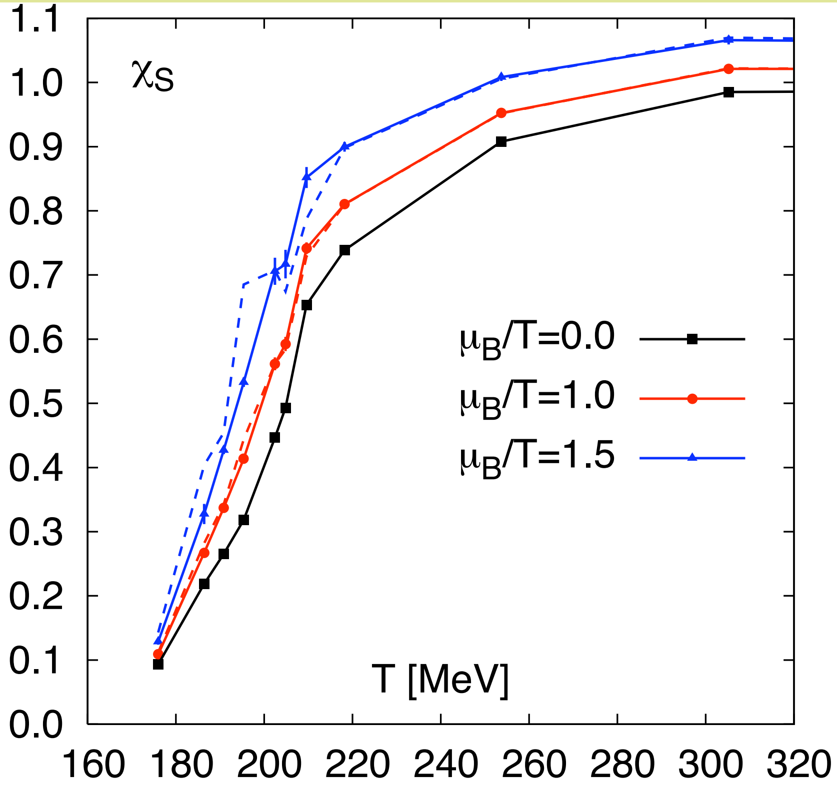
baryon number - strangeness correlations

strangeness fluctuations

$$\chi_S = 2c_{0,2}^{B,S} + 2c_{2,2}^{B,S} \left(\frac{\mu_B}{T}\right)^2 + \cdots$$







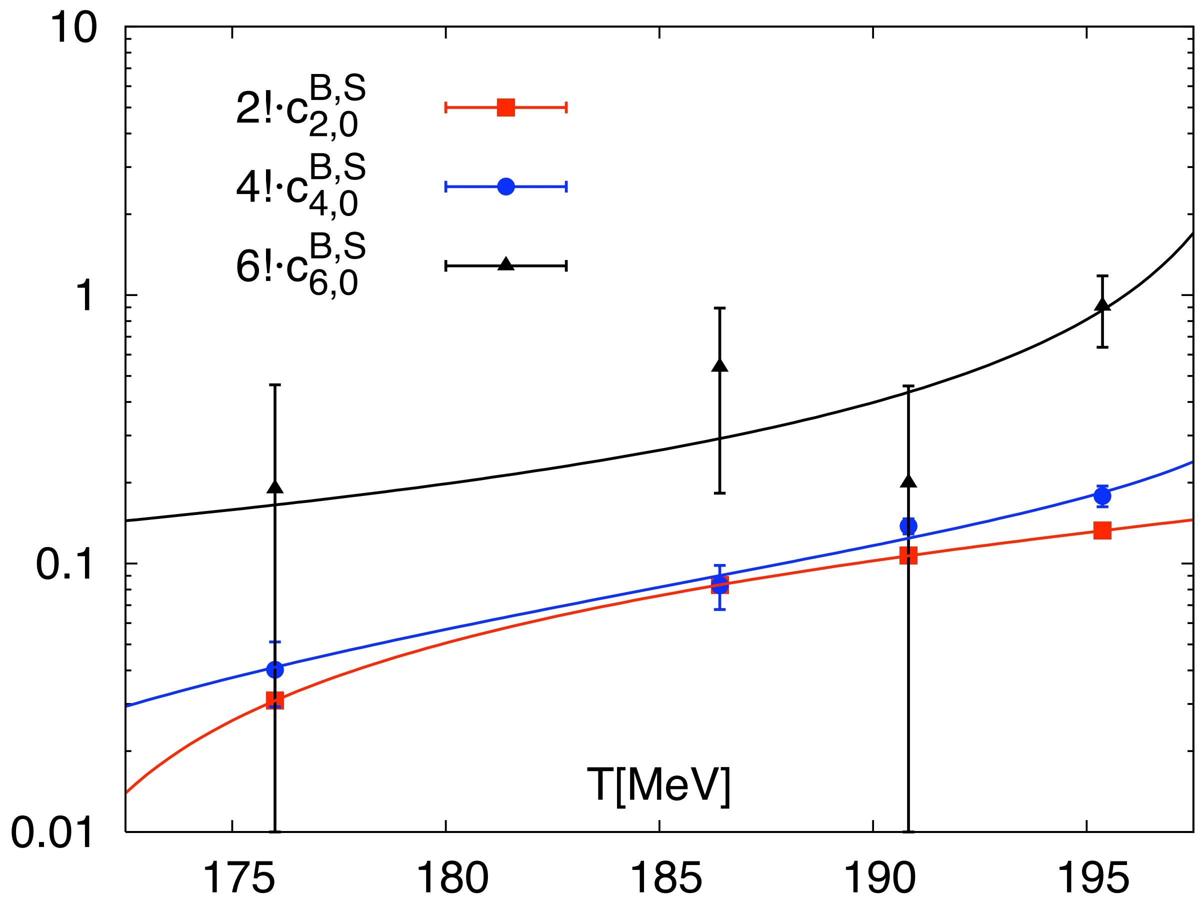
$$C_{BS} = \frac{c_{1,1}^{B,S} + 3c_{3,1}^{B,S}\left(\frac{\mu_B}{T}\right)^2 + \cdots}{\chi_S(\frac{\mu_B}{T})}$$

$\longrightarrow$ LO introduces a peak in the fluctuations/correlations, NLO shifts the peak towards smaller temperatures

$\longrightarrow$ truncation errors become large at $\mu_B/T \gtrsim 1.5$

The critical line (chiral limit)

combined fit to c_2, c_4, c_6



scaling field (chiral limit):

$$t = \frac{T - T_c}{T_c} + \kappa \mu_B^2$$

free energy:

$$f = A_\pm |t|^{2-\alpha} + \text{regular}$$

critical line:

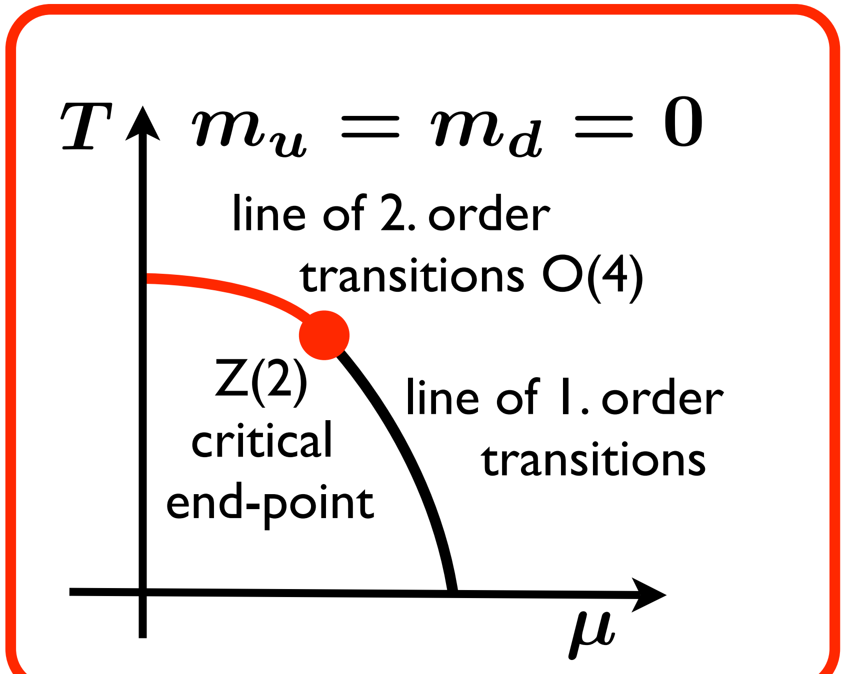
$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \mu_B^2$$

$$2! \cdot c^{2,0}_{B,S} \sim \mp 2A_\pm (2-\alpha)\kappa |t|^{1-\alpha} + b_2 t + c_2\ ;$$

$$4! \cdot c^{4,0}_{B,S} \sim -12A_\pm (2-\alpha)(1-\alpha)\kappa^2 |t|^{-\alpha} + b_4 t + c_4\ ;$$

$$6! \cdot c^{6,0}_{B,S} \sim \pm 120A_\pm (2-\alpha)(1-\alpha)(-\alpha)\kappa^3 |t|^{-1-\alpha}\ .$$

expected phase diagram

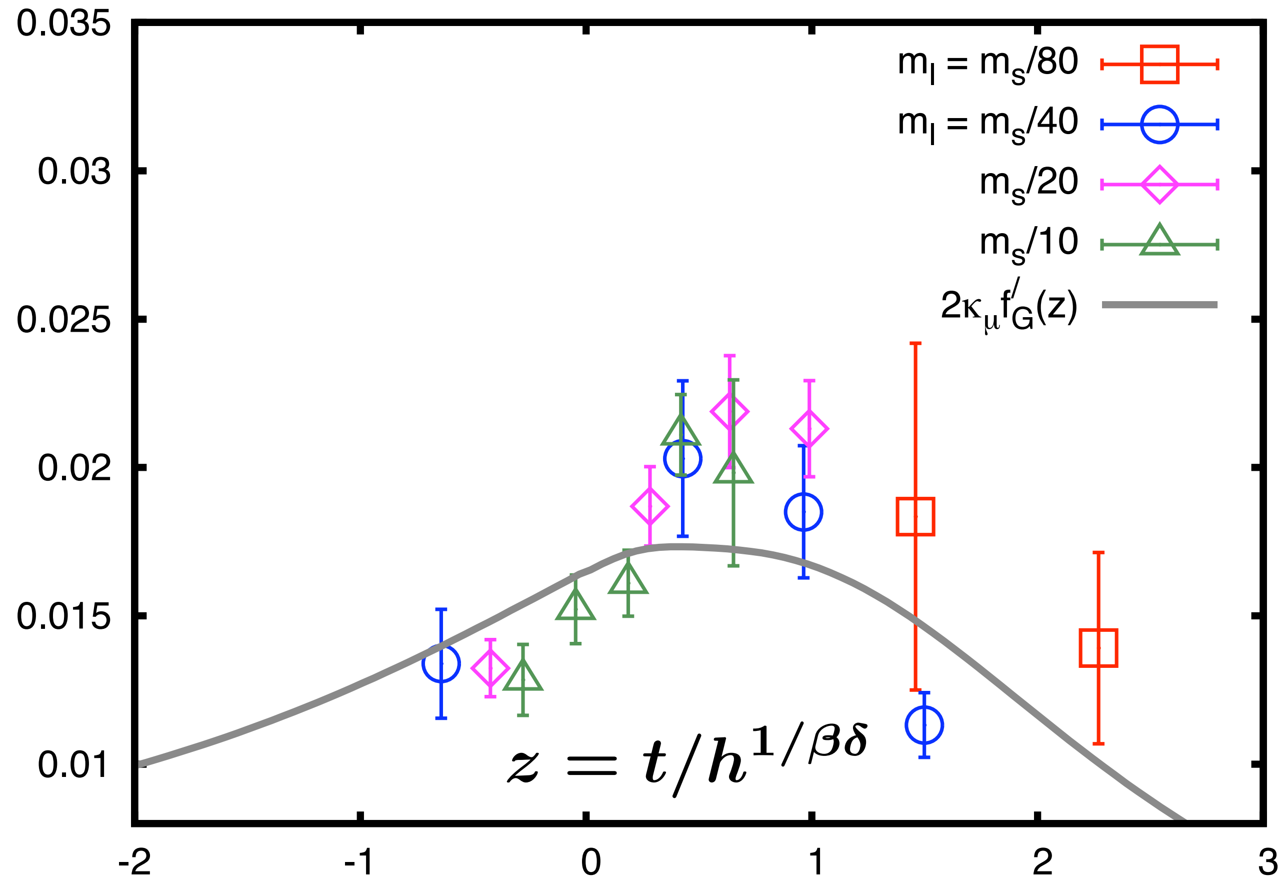


$\longrightarrow$ coefficients c_2, c_4 dominated by regular part

$\longrightarrow$ will work better with $c_2^{\psi\bar\psi}, \ldots$

The critical line (chiral limit)

fit to $c_2^{\bar\psi\psi}$



courtesy S.Mukherjee

scaling field (chiral limit):

$$t = \frac{T - T_c}{T_c} + \kappa\mu_B^2$$

magnetic EoS:

$$M = h^{1/\delta} f_G(t/h^{1/\beta\delta})$$

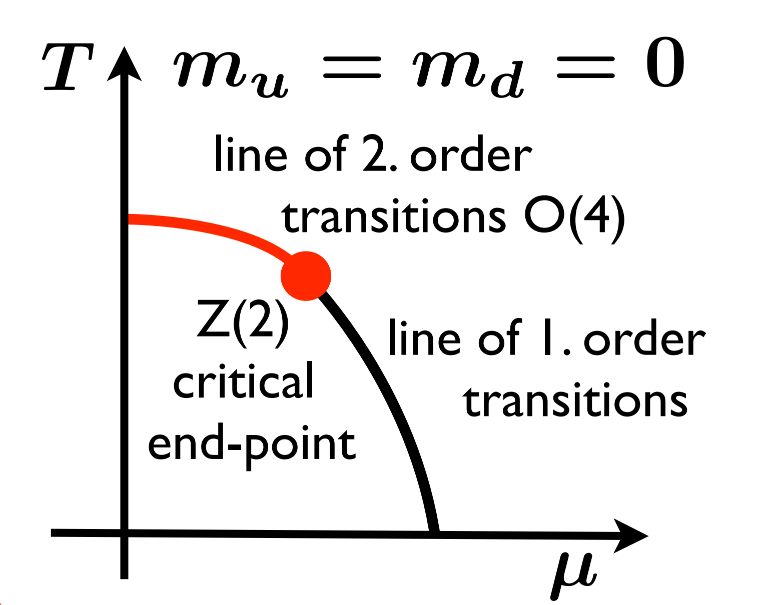
critical line:

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa\mu_B^2$$

$$-h^{(1-\beta)/\beta\delta} c_2^{\bar\psi\psi} t_0 m_s T^{-1} = 2\kappa f'_G(z)$$

only one fit-parameter

expected phase diagram



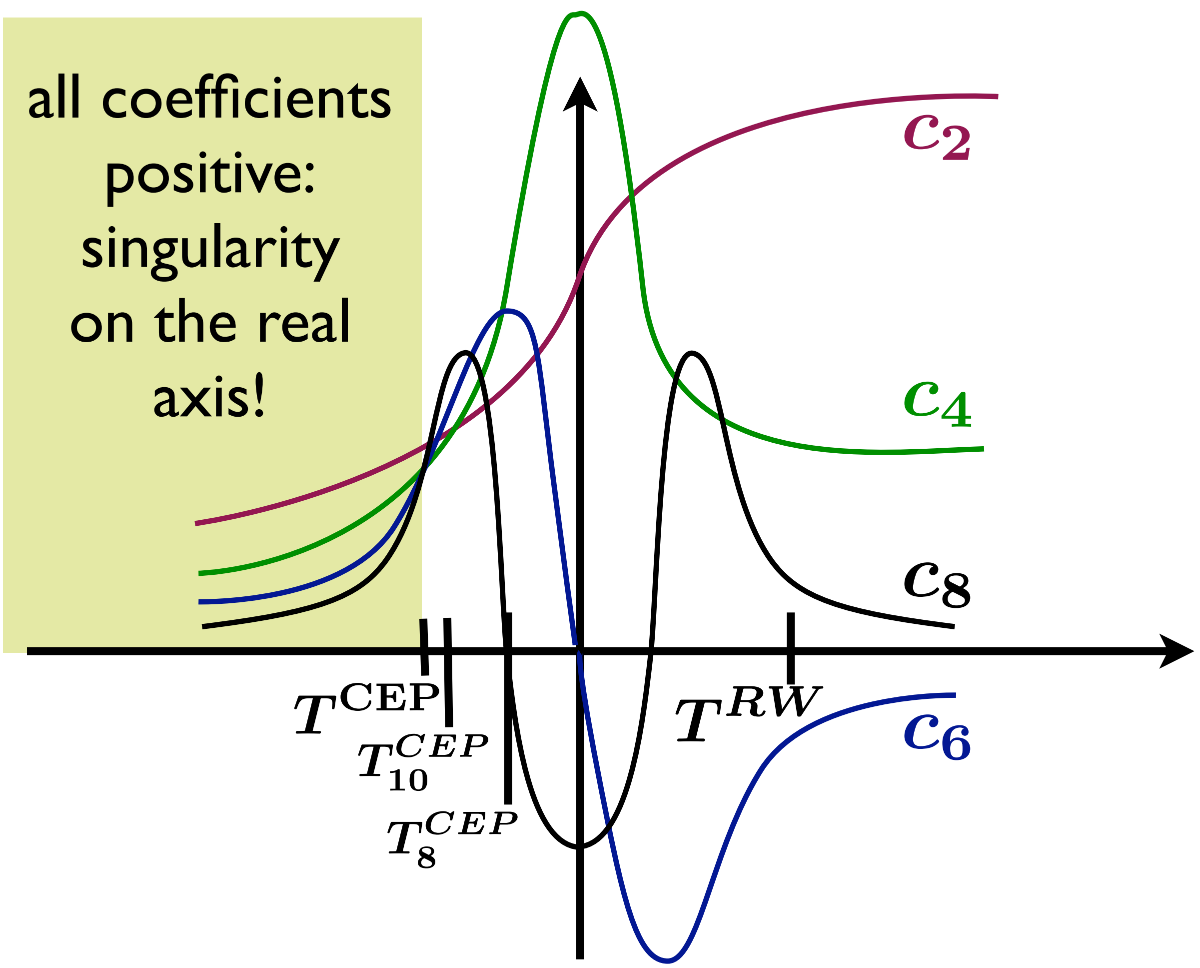
The critical endpoint (I)

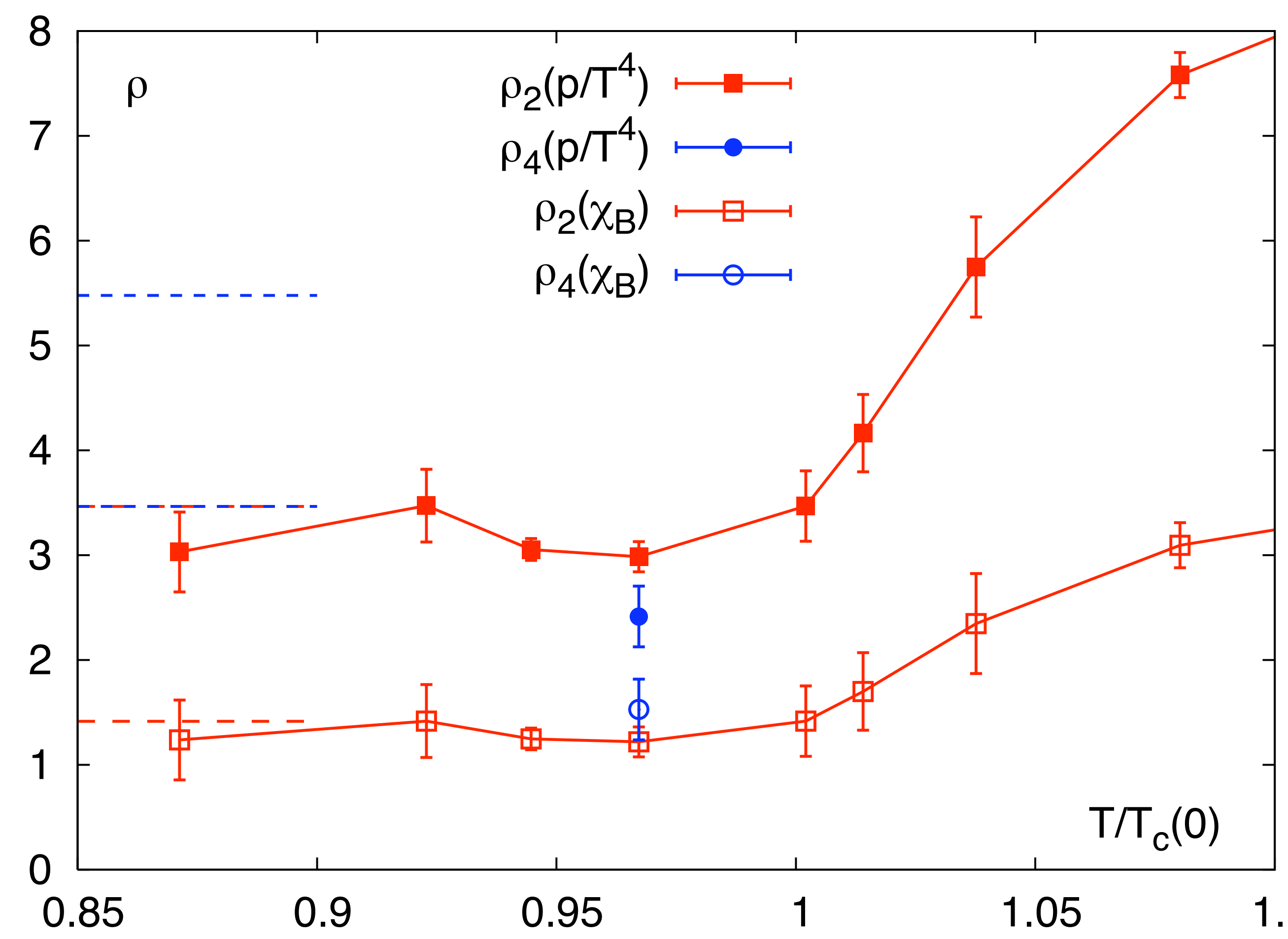
<u>method for locating of the CEP:</u>

- determine largest temperature where all coefficients are positive $\rightarrow T^{\text{CEP}}$
- determine the radius of convergence at this temperature $\rightarrow \mu^{\text{CEP}}$

$$p = c_0 + c_2 \left(\mu_B/T\right)^2 + c_4 \left(\mu_B/T\right)^4 + \cdots$$

$$\chi_B = 2c_2 + 12c_4 \left(\mu_B/T\right)^2 + 30c_6 \left(\mu_B/T\right)^4 + \cdots$$





first non-trivial estimate of T^{CEP} by c_8
second non-trivial estimate of T^{CEP} by c_{10}

$$\rho_n(p) = \sqrt{c_n/c_{n+2}}$$

$$\rho = \lim_{n\to\infty} \rho_n$$

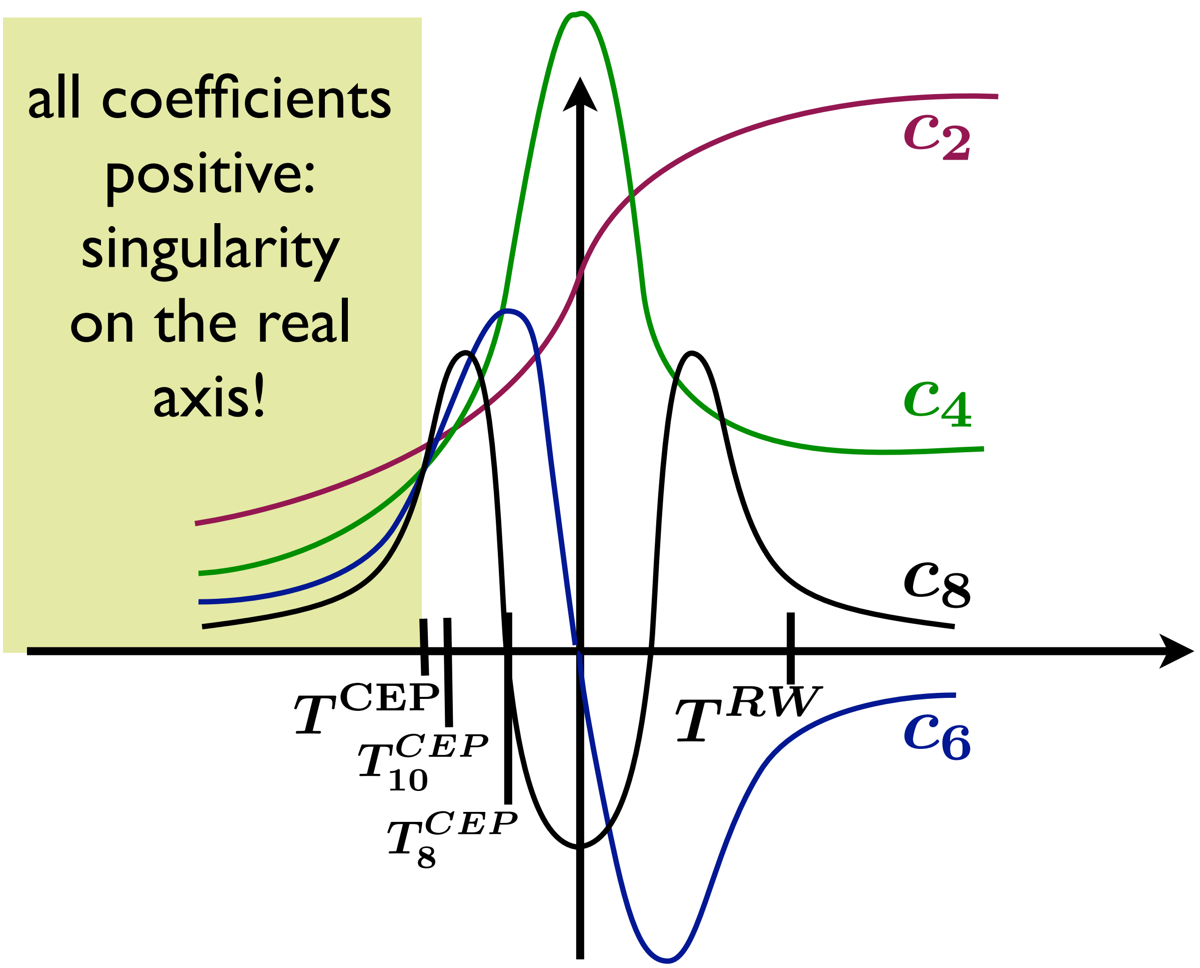
The critical endpoint (II)

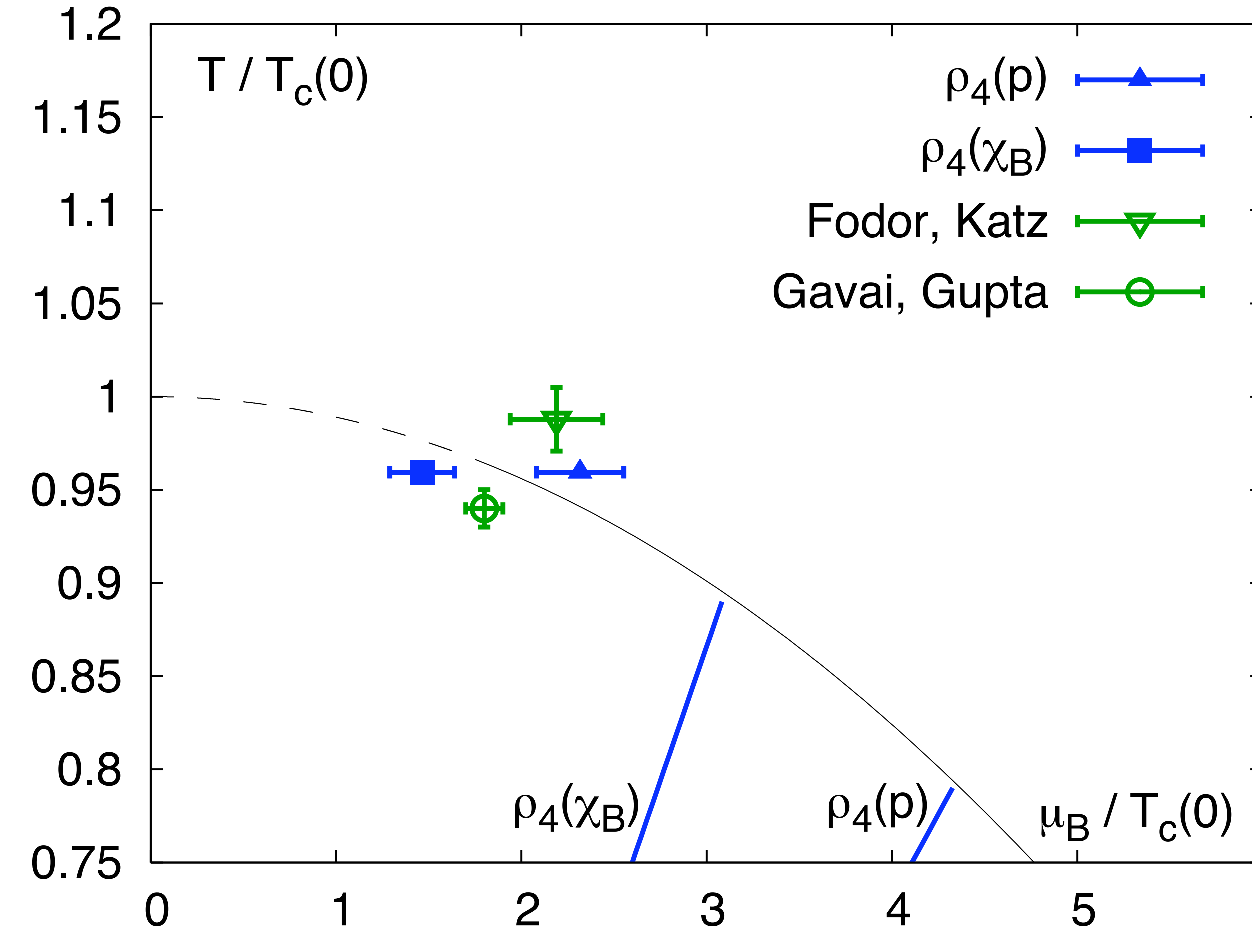
method for locating of the CEP:

- determine largest temperature where all coefficients are positive ➔ $T^{\rm CEP}$
- determine the radius of convergence at this temperature ➔ $\mu^{\rm CEP}$

$$p = c_0 + c_2\left(\mu_B/T\right)^2 + c_4\left(\mu_B/T\right)^4 + \cdots$$

$$\chi_B = 2c_2 + 12c_4\left(\mu_B/T\right)^2 + 30c_6\left(\mu_B/T\right)^4 + \cdots$$





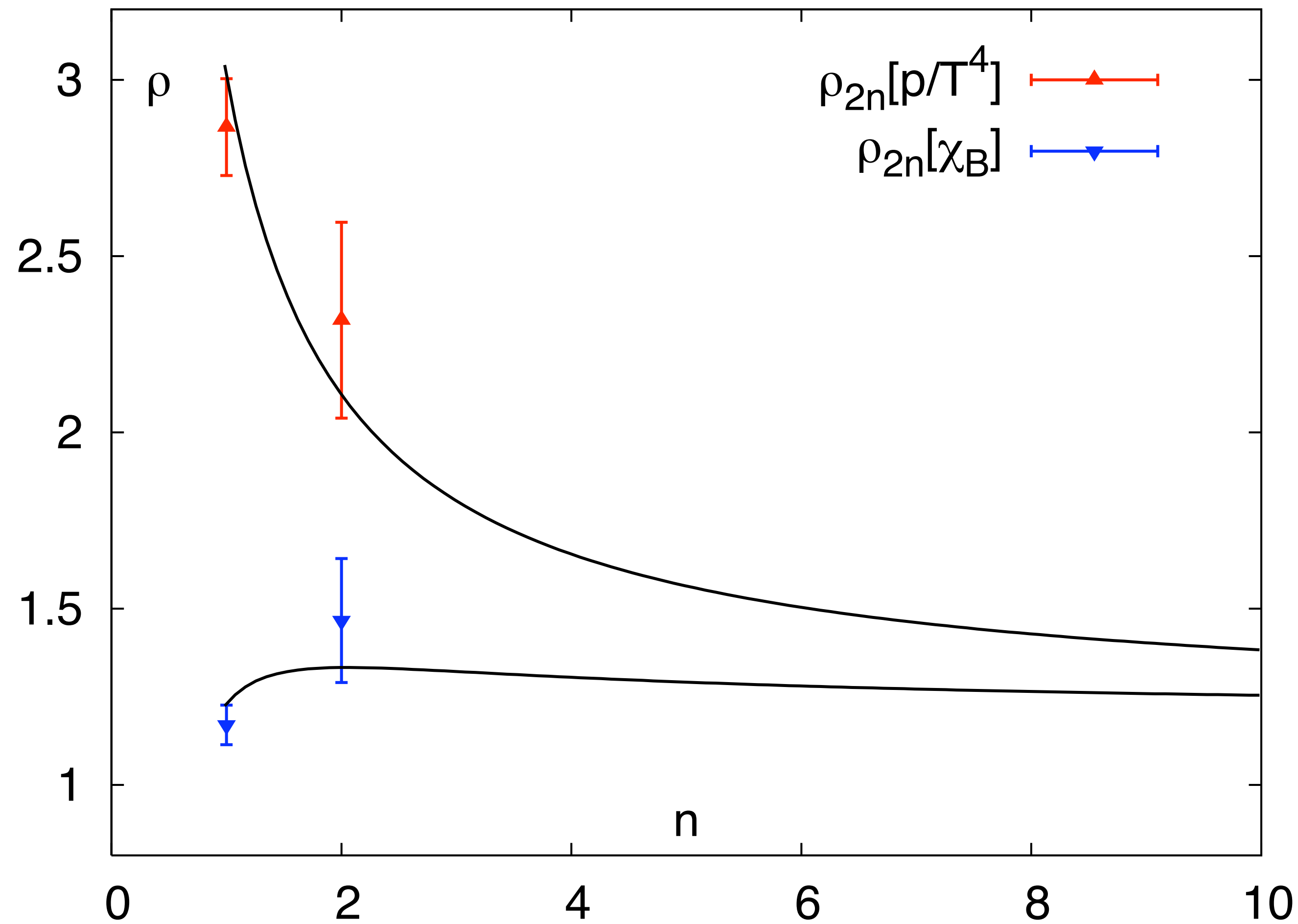
first non-trivial estimate of $T^{\rm CEP}$ by c_8
second non-trivial estimate of $T^{\rm CEP}$ by c_{10}

$$\rho_n(p) = \sqrt{c_n/c_{n+2}}$$

$$\rho = \lim_{n\to\infty} \rho_n$$

The critical endpoint (III)

What is the asymptotic behavior of ρ_n ?



$$\rho_n(p) \sim \rho\left(1 + \frac{3-\alpha}{2n}\right)$$
$$\rho_n(\chi_B) \sim \rho\left(1 + \frac{1-\alpha}{2n}\right)$$

$\longrightarrow$ $\rho_n(\chi_B)$ more stable

$\longrightarrow$ still not the correct critical point!

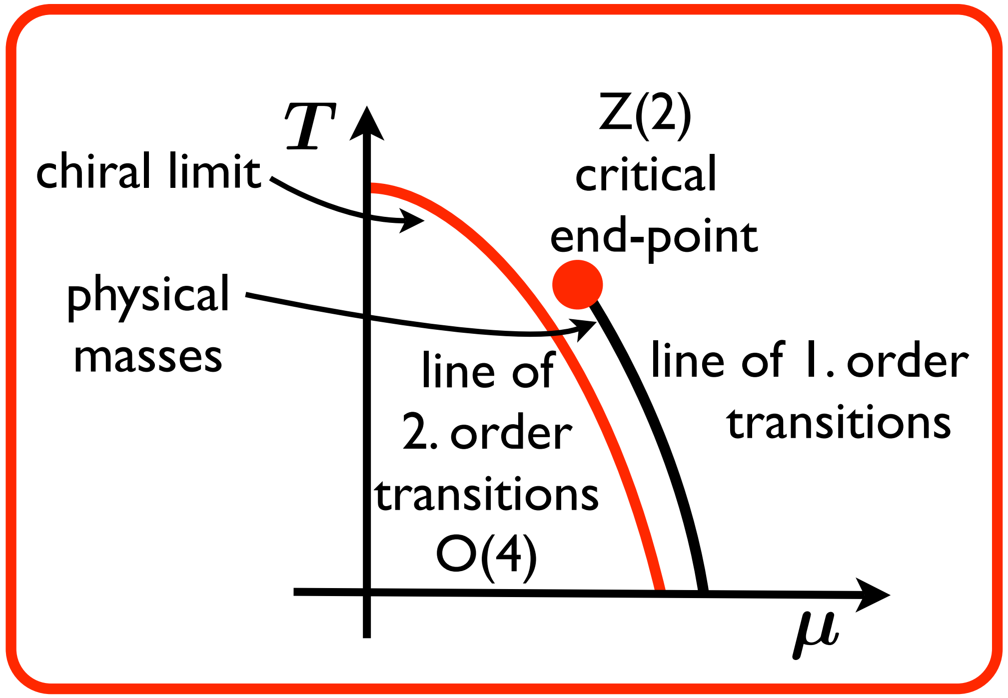
scaling field:

$$t = \frac{T - T_c(\mu_c)}{T_c(\mu_c)} + \kappa'\left(\mu_B^2 - \mu_c^2\right)$$

free energy:

$$f = A_\pm |t|^{2-\alpha} + \text{regular}$$

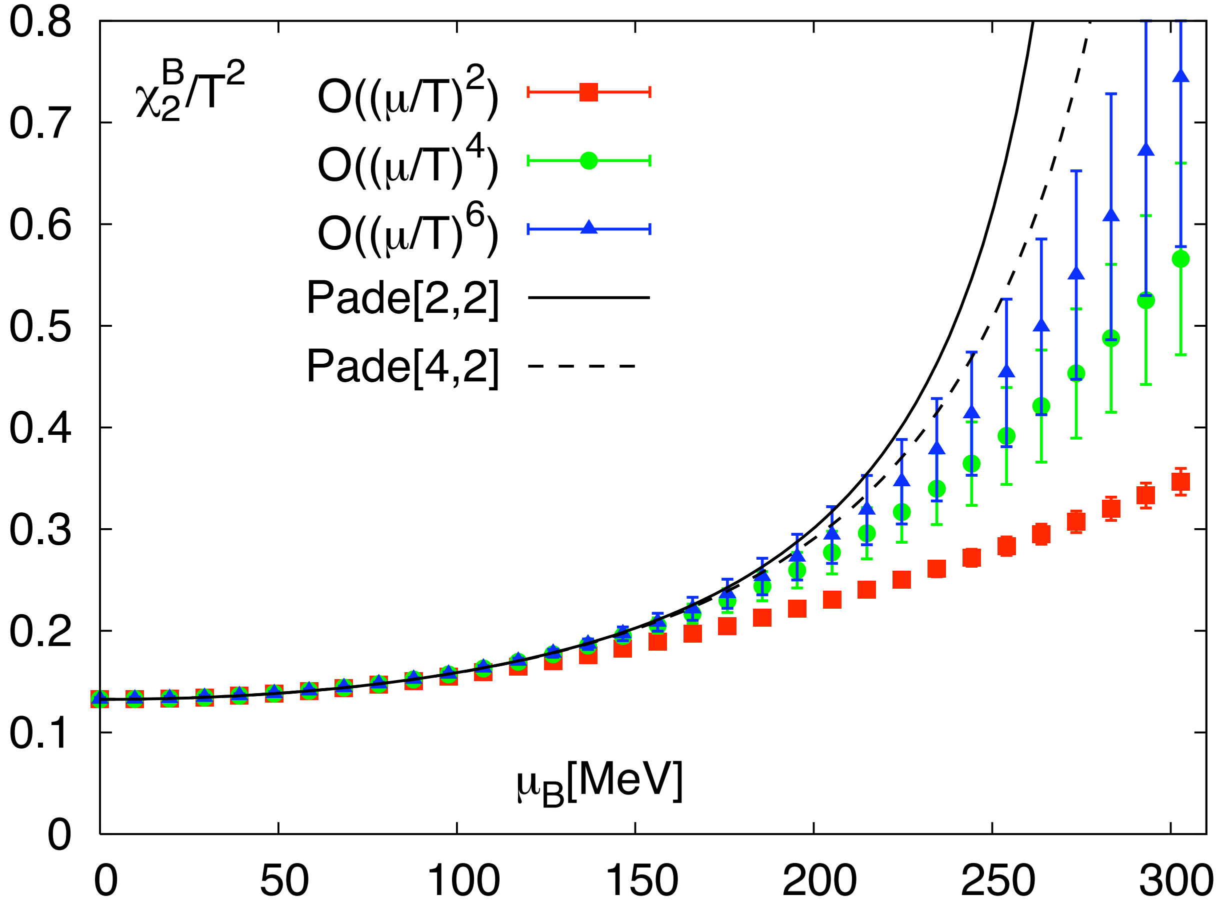
expected phase diagram

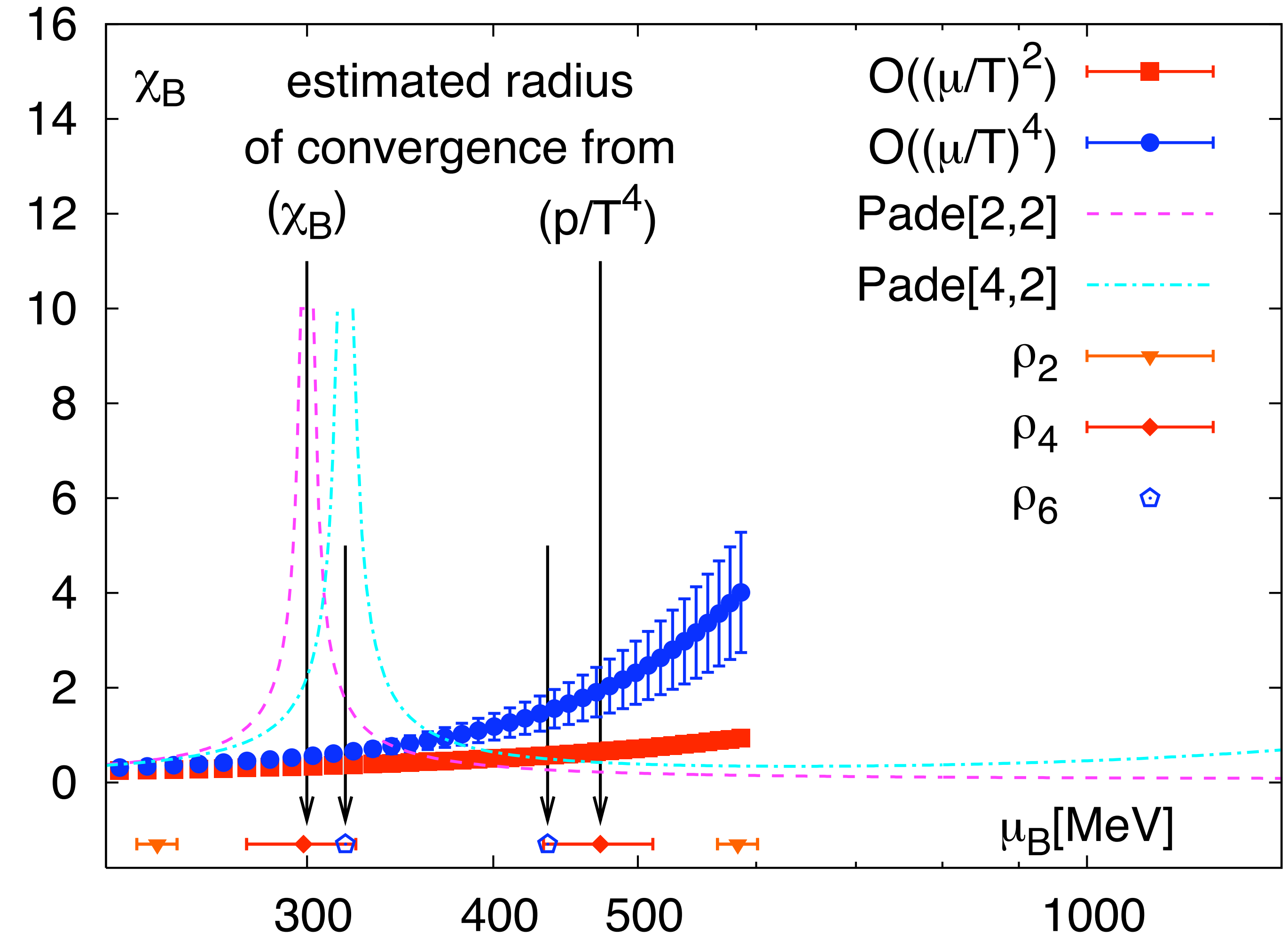


Taylor vs. Pade

Pade [2,2]:

$$\frac{\chi_B}{T^2} = \frac{2c_2c_4 + (12c_4^2 - 5c_2c_6)\left(\frac{\mu_B}{T}\right)^2}{c_4 - \frac{5}{2}c_6\left(\frac{\mu_B}{T}\right)^2}$$





→ good agreement of Taylor and Pade for $\mu_B/T \lesssim 1$

→ Pade approximants [2,2] and [4,2] have a pole at ρ_4 and ρ_6, respectively

fixing c_8 by the asymptotic behavior of Pade[4,2], demanding:

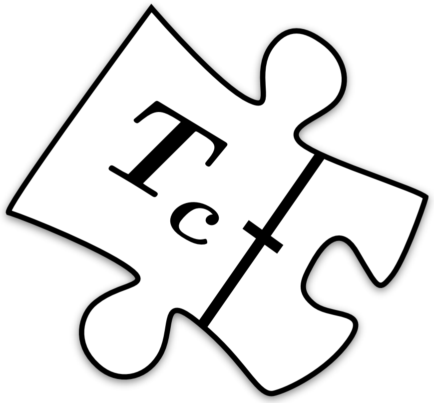
$$\lim_{\mu_B \to \infty} \chi_B \approx \frac{1}{3} + \frac{1}{9\pi^2}\left(\frac{\mu_B}{T}\right)^2$$

Summary

- Universal scaling behavior is observed near the chiral limit of (2+1)-flavor QCD
- Hints for the physical point being in the attraction region of a second order critical point in the chiral limt
- A Taylor expansion of the pressure is used to obtain lattice QCD results at nonzero density and, in addition, provides a method to locate the CEP.
- Fluctuations and correlations are well described by a free gas of quarks above T>(1.5-1.7)Tc and by a resonance gas for T<Tc.
- Truncation errors of the Taylor series becomes large for $\mu_B/T \gtrsim (1-1.5)$
- Expansion coefficients of the chiral condensate can be used to obtain the curvature of the critical line in the chiral limit
- Taylor expansion coefficients can be used to construct Pade approximants that asymptotically approach the free gas behavior.

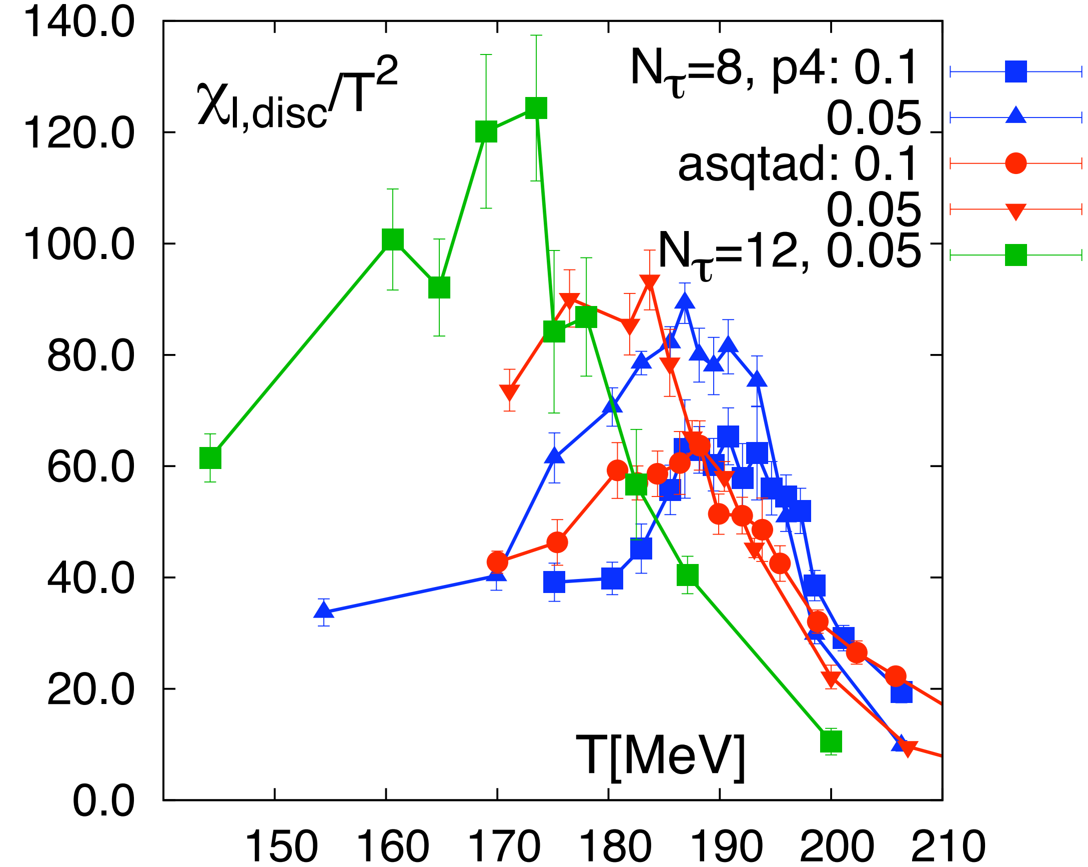
The critical temperature at $\mu_B = 0$

first peace of the puzzle:



(see talk by M. Cheng)

disconnected chiral susceptibility



HotQCD preliminary

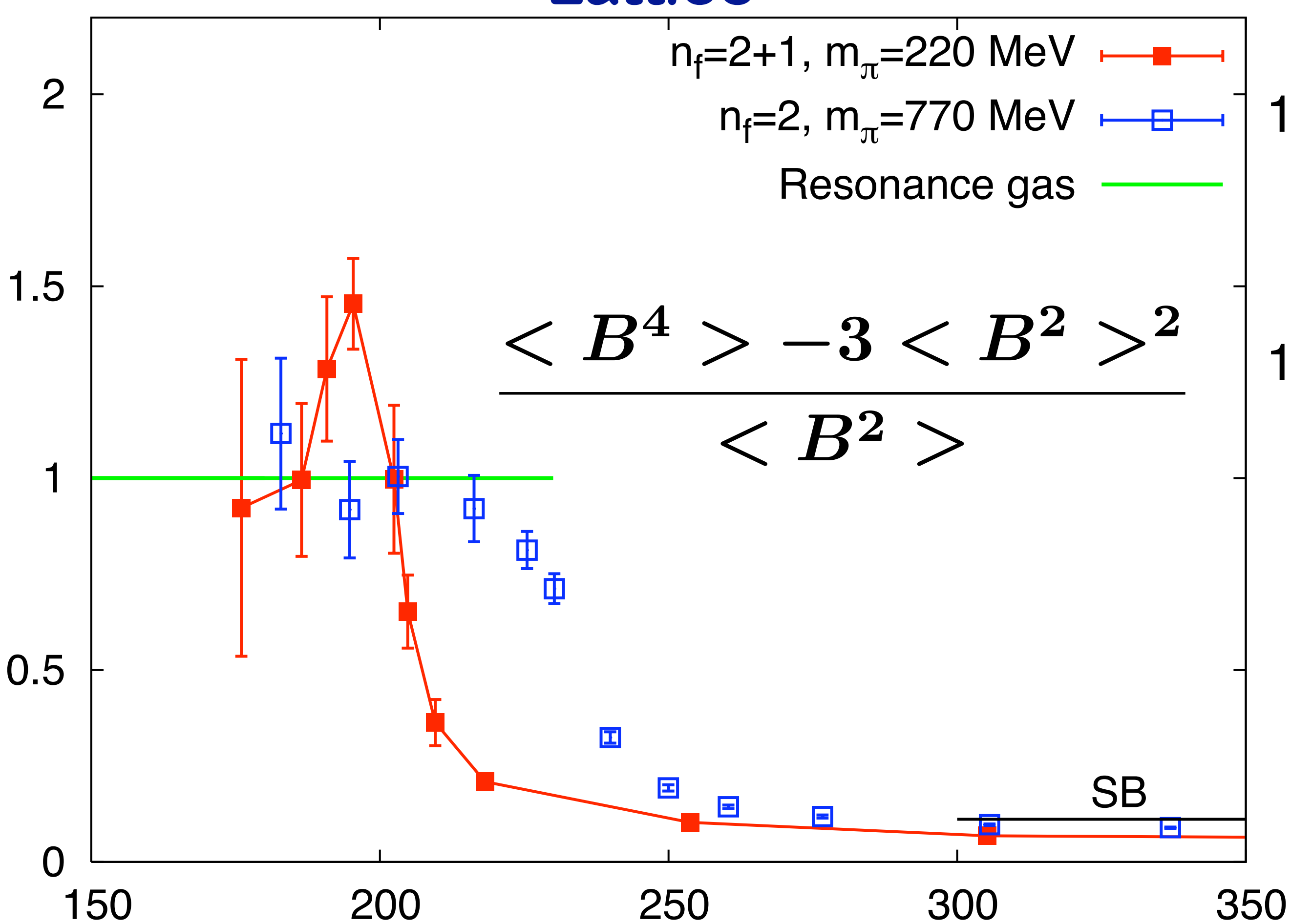
$N_\tau = 8, m_l/m_s = 1/20$:
$T_c \approx (180 - 190)$ MeV

$N_\tau = 12$: suggests continuum extra-polated value < 170 MeV

Hadronic fluctuations (II)

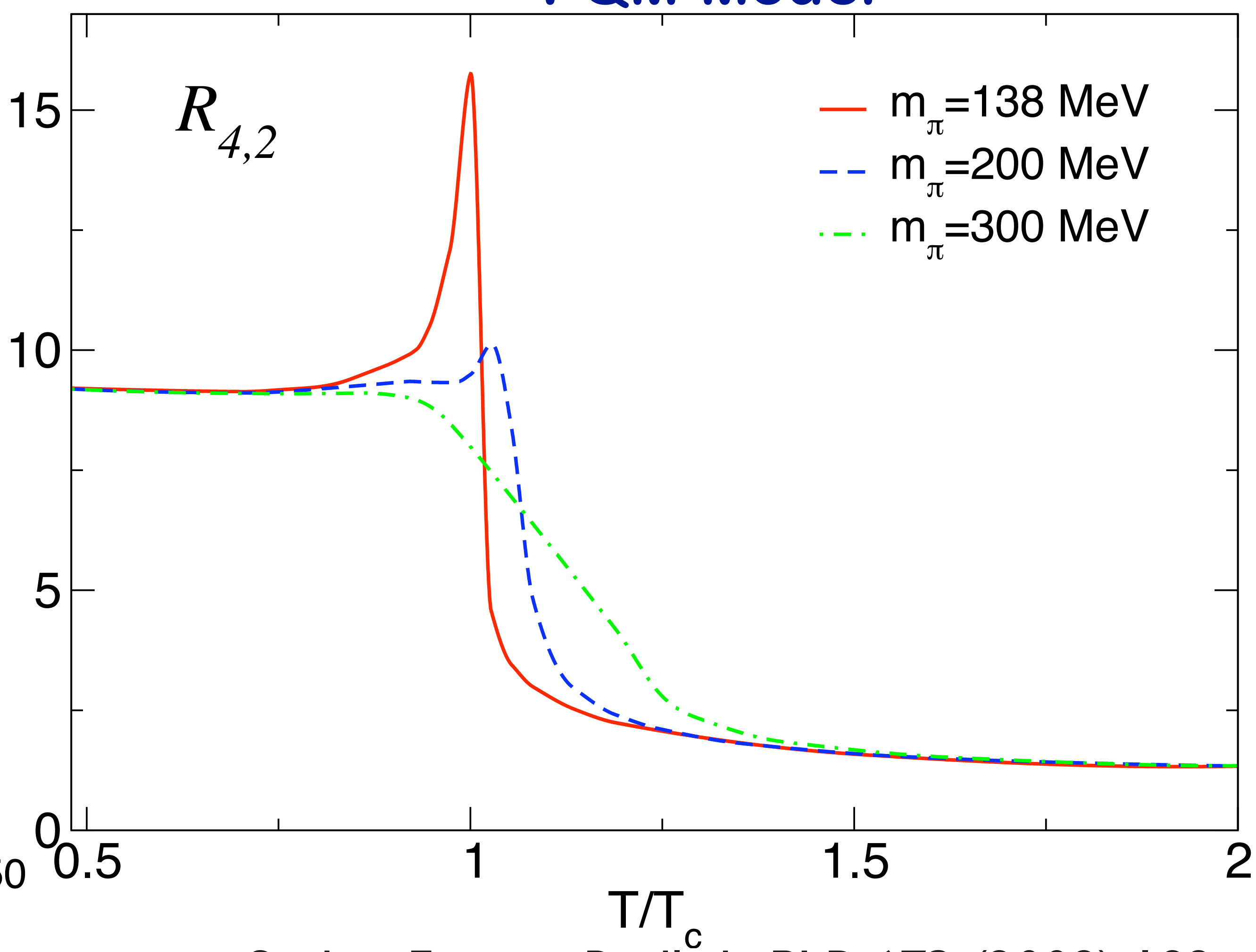
B-Kurtosis (c_4/c_2)

Lattice



red: CS, J.Phys.G35 (2008) 104093.
blue: Allton et all., Phys. Rev. D71 (2005) 054508.

PQM-Model



Stokic, Friman, Redlich, PLB 673 (2009) 192.

→ fluctuations increase over resonance gas level?

→ **good experimental observable?**

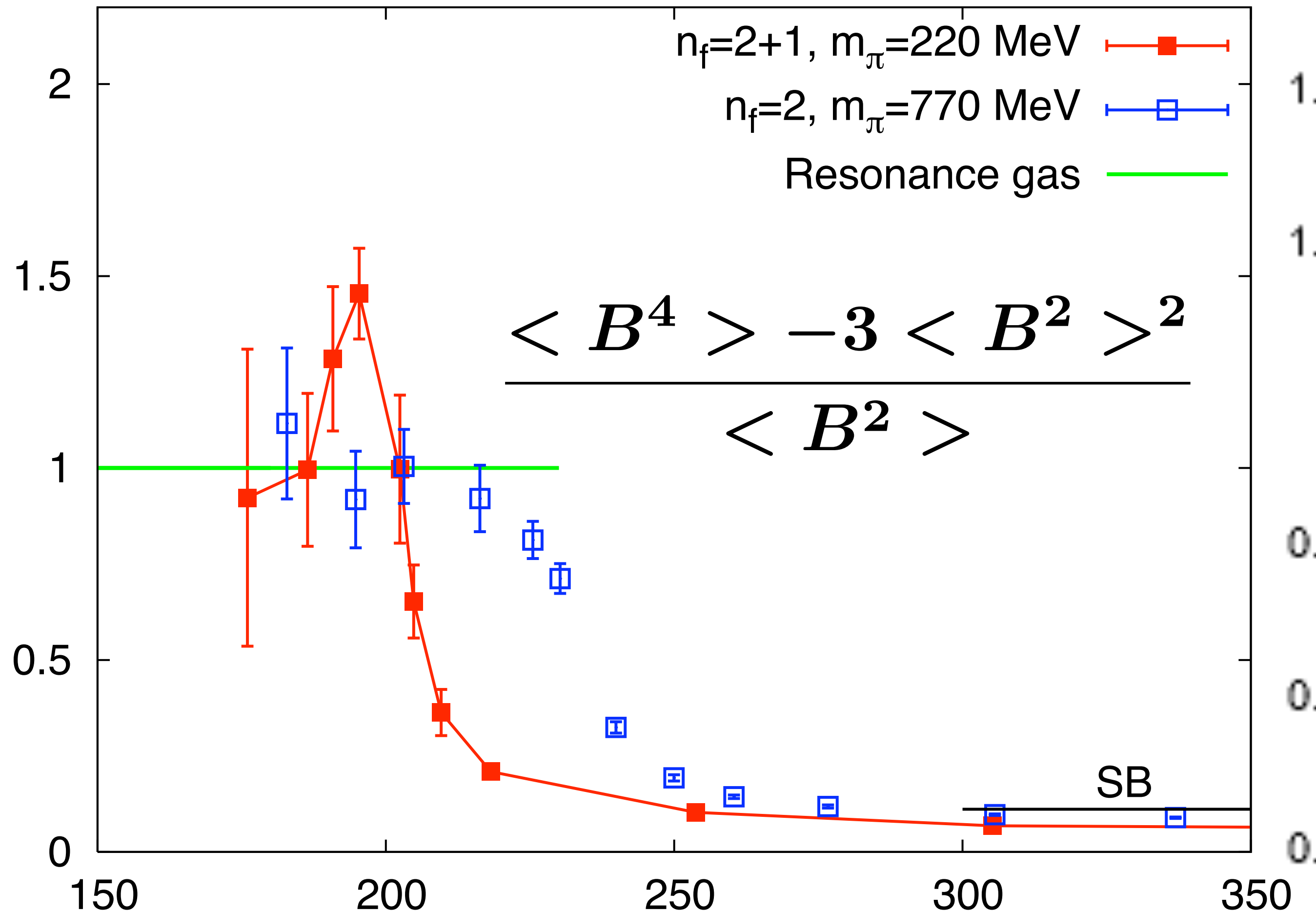
chiral limit:

$$\chi_4^B, \chi_4^Q \propto |T - T_c|^{-\alpha} \text{ + regular}$$

Hadronic fluctuations (II)

B-Kurtosis (c_4/c_2)

Lattice



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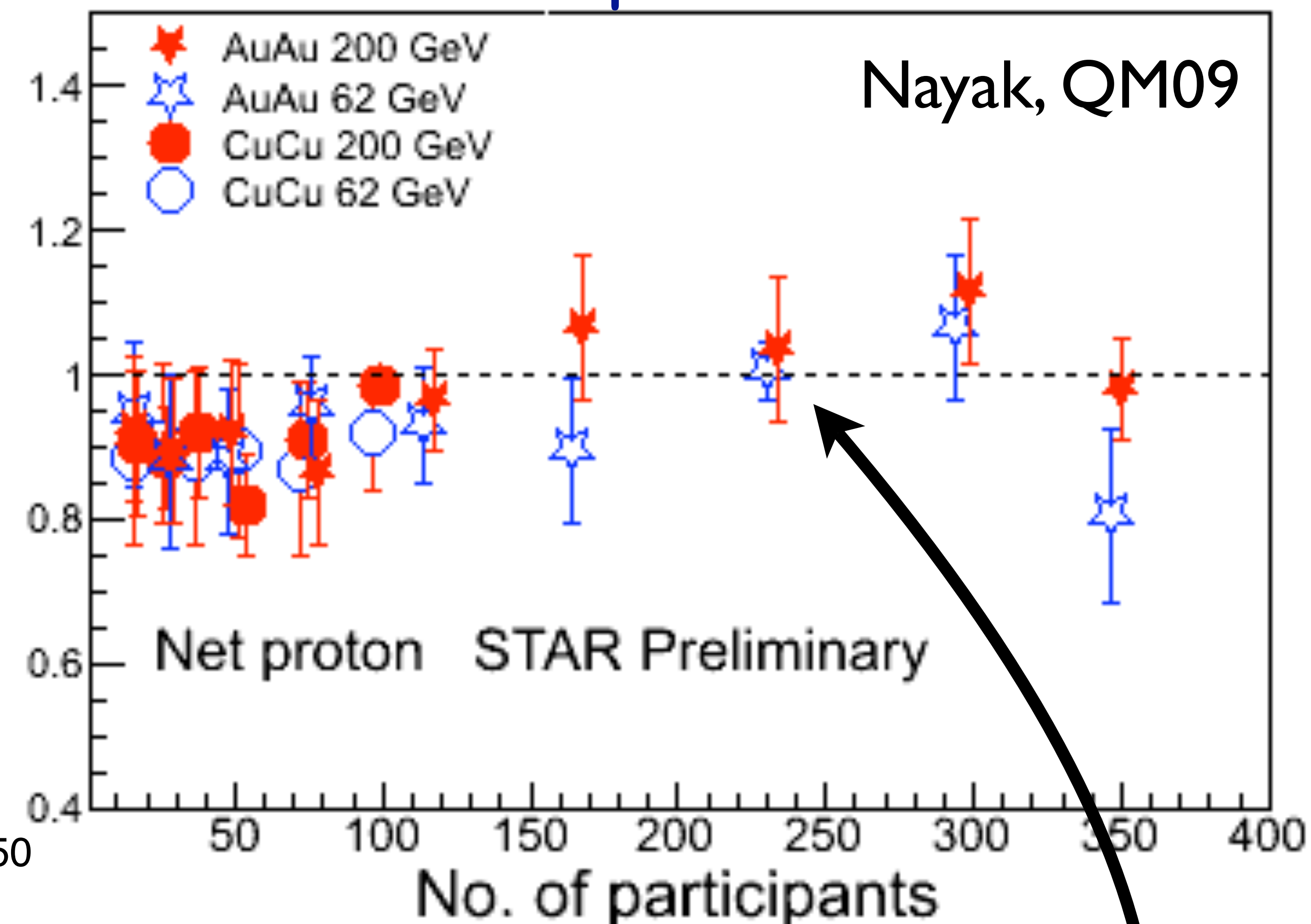
Experiment



→ fluctuations increase over resonance gas level?

→ **good experimental observable?**

only Gaussian fluctuations in protons observed

chiraler Limes:

$$\chi_4^B, \chi_4^Q \propto |T - T_c|^{-\alpha} + \text{regular}$$