In Search of the Perfect Fluid

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See T. Schäfer, D. Teaney, “Perfect Fluidity” [arXiv:0904.3107]

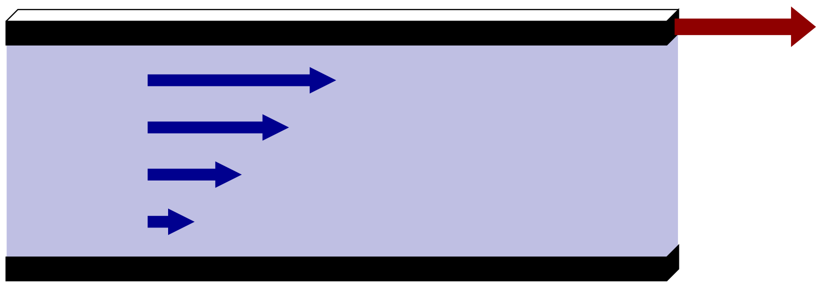
The bottom-line

Remarkably, the best fluids that have been observed are the *coldest* and the *hottest* fluid ever created in the laboratory, cold atomic gases (10^{-6}K) and the quark gluon plasma (10^{12}K) at RHIC.

Both of these fluids come close to a bound on the shear viscosity that was first proposed based on calculations in string theory, involving non-equilibrium evolution of back holes in 5 (and more) dimensions.

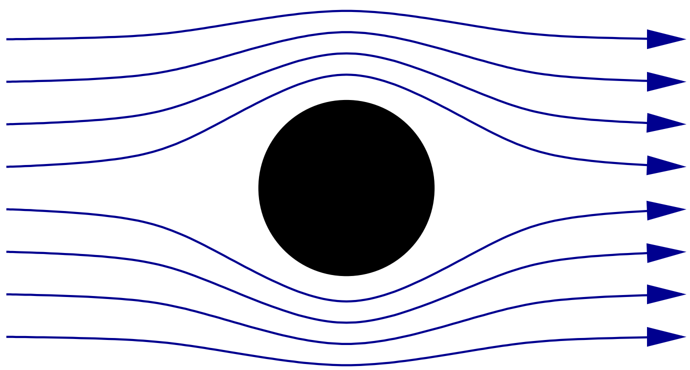
Measures of Perfection

Viscosity determines shear stress ("friction") in fluid flow



$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

Dimensionless measure of shear stress: Reynolds number



$$Re \quad = \quad \frac{n}{\eta} \quad \times \quad mvr$$

fluid property — flow property

- $[\eta/n] = \hbar$
- Relativistic systems $Re = \dfrac{s}{\eta} \times \tau T$

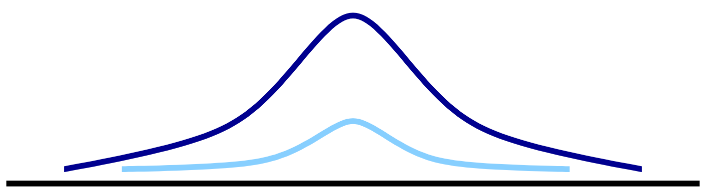
Other sources of dissipation (thermal conductivity, bulk viscosity, ...) vanish for certain fluids, but shear viscosity is always non-zero.

There are reasons to believe that η is bounded from below by a constant times $\hbar s/k_B$. In a large class of theories $\eta/s \geq \hbar/(4\pi k_B)$.

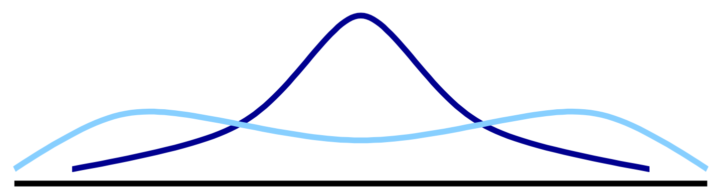
A fluid that saturates the bound is a "perfect fluid".

Fluids: Gases, Liquids, Plasmas, ...

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



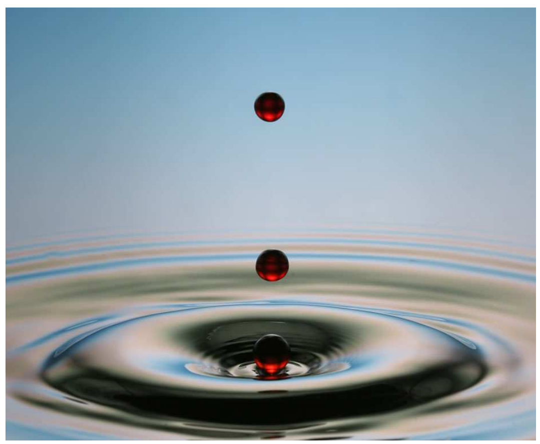
$\tau \sim \tau_{micro}$



$\tau \sim \lambda^{-1}$

Historically: Water

$(\rho, \epsilon, \vec{\pi})$



Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \vec{j}^{\epsilon} = 0$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0$$

Constitutive relations: Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right) + O(\partial^2)$$

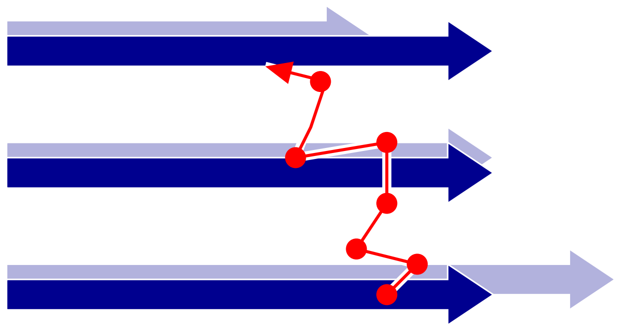
reactive dissipative 2nd order

Measure of perfection: $\frac{\delta \Pi_{ij}}{\Pi_{ij}} \sim \frac{\eta(\partial \cdot v)}{P} \sim \frac{\eta}{n}\frac{1}{\tau T}$

Kinetic Theory

Kinetic theory: conserved quantities carried by quasi-particles

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p + \vec{F}\cdot\vec{\nabla}_p f_p = C[f_p]$$



$$\eta \sim \frac{1}{3} n\,\bar{p}\,l_{mfp}$$

Normalize to density. Uncertainty relation suggests

$$\frac{\eta}{n} \sim \bar{p}\,l_{mfp} \geq \hbar$$

Also: $s \sim k_B n$ and $\eta/s \geq \hbar/k_B$

Validity of kinetic theory as $\bar{p}\,l_{mfp} \sim \hbar$?

Effective Theories for Fluids (Here: Weak Coupling QCD)

$$\mathcal{L} = \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}$$

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p = C[f_p] \qquad (\omega < T)$$

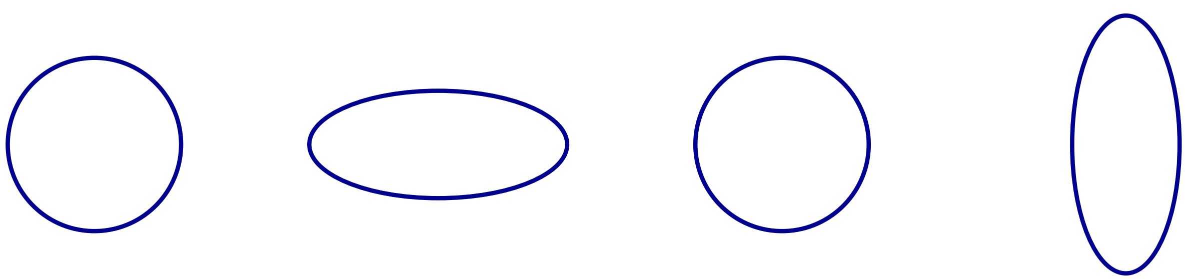
$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \qquad (\omega < g^4 T)$$

Holographic Duals: Transport Properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT temperature	$\Leftrightarrow$	Hawking temperature
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon
shear viscosity	$\Leftrightarrow$	Graviton absorption cross section $\sim$ area of event horizon

$$T_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} \qquad g_{\mu\nu} = g^0_{\mu\nu} + \gamma_{\mu\nu}$$



Holographic Duals: Transport Properties

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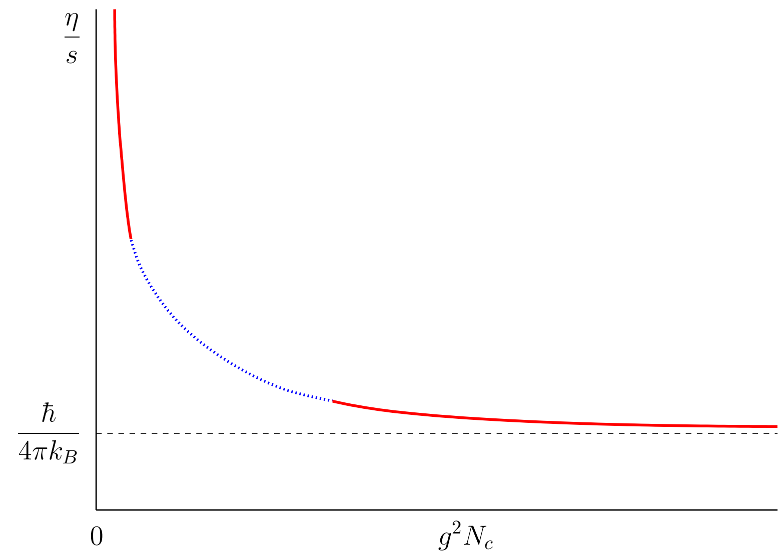
Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Son and Starinets (2001)



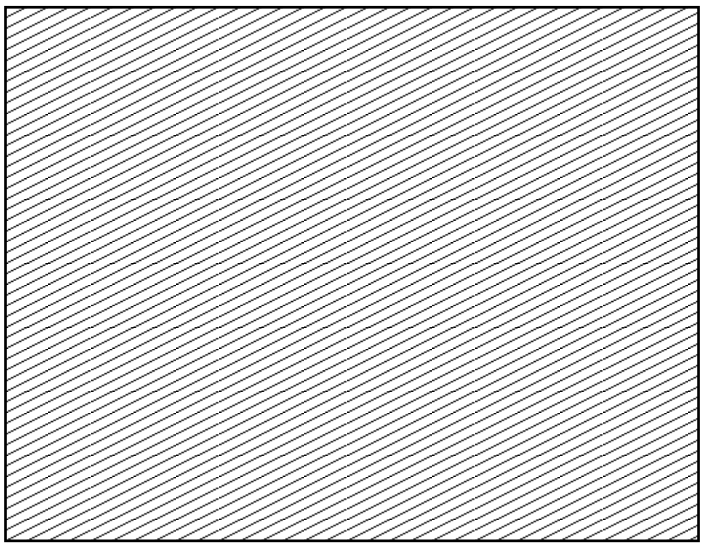
Strong coupling limit universal? Provides lower bound for all theories?

Effective Theories (Strong coupling)

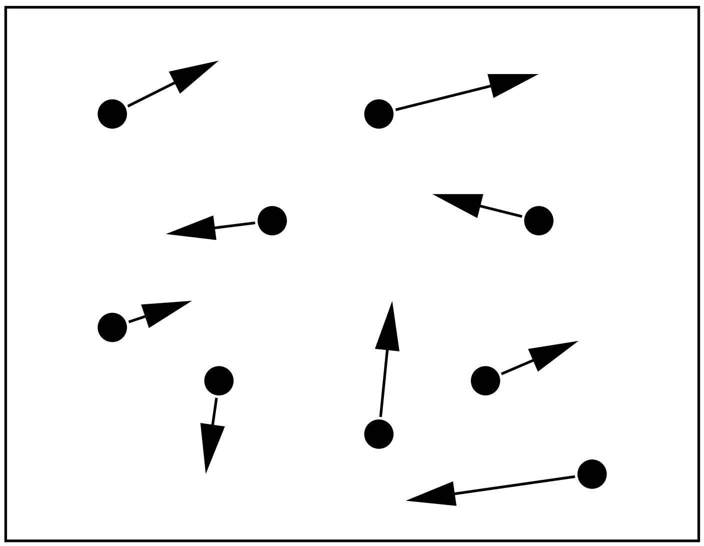
$$\mathcal{L} = \bar{\lambda}(i\sigma \cdot D)\lambda - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \ldots \;\Leftrightarrow\; S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\mathcal{R} + \ldots$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \quad (\omega < T)$$

Kinetics vs No-Kinetics



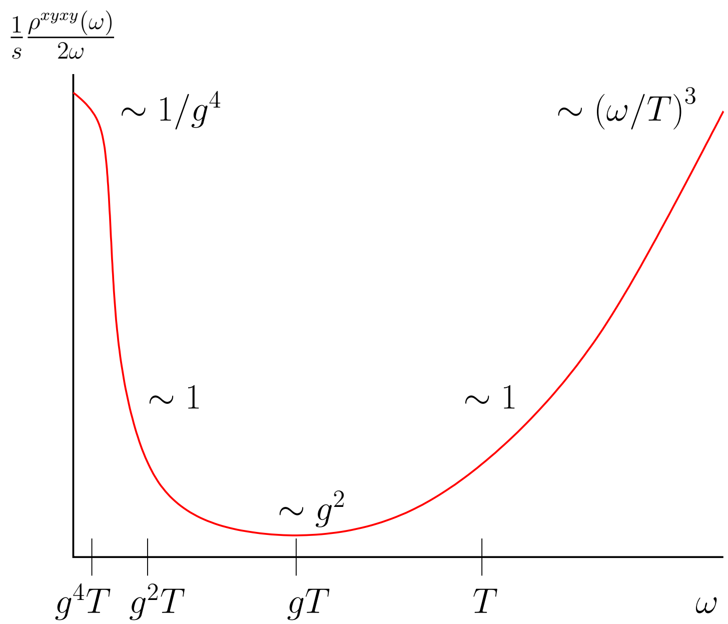
AdS/CFT low viscosity goo



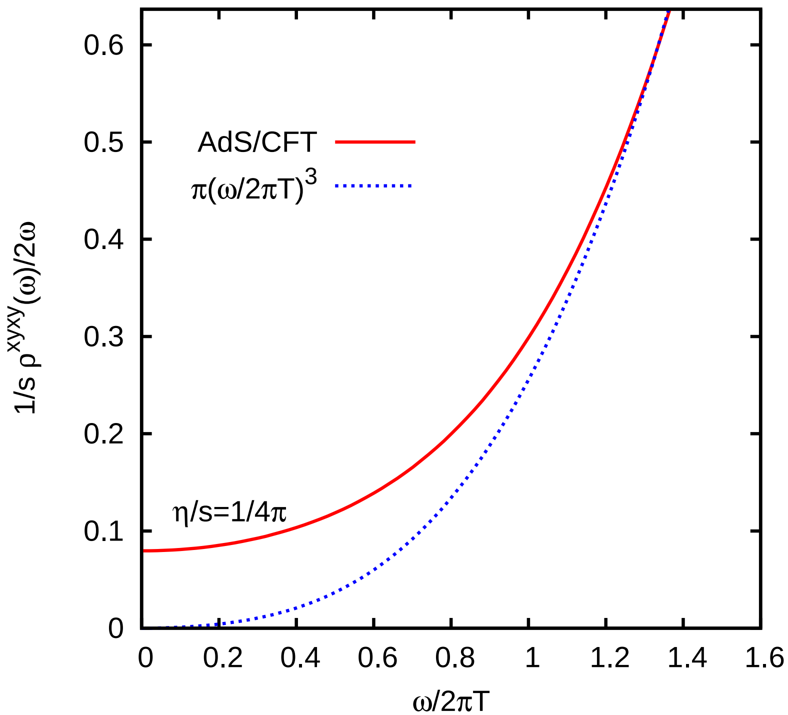
pQCD kinetic plasma

Kinetics vs No-Kinetics

Spectral function $\rho(\omega) = \mathrm{Im} G_R(\omega, 0)$ associated with T_{xy}



weak coupling QCD



strong coupling AdS/CFT

transport peak vs no transport peak

Perfect Fluids: How to be a contender?

Bound is quantum mechanical

need quantum fluids

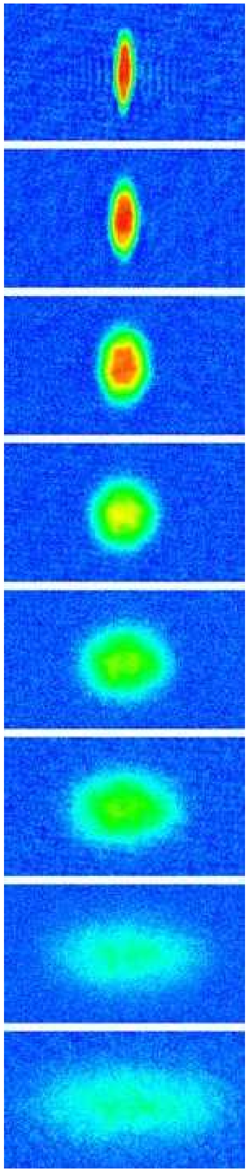
Bound is incompatible with weak coupling and kinetic theory

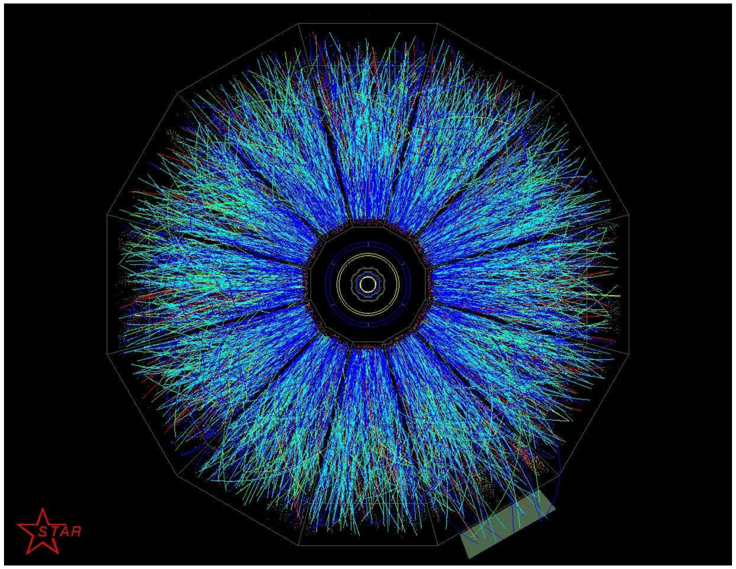
strong interactions, no quasi-particles

Model system has conformal invariance (essential?)

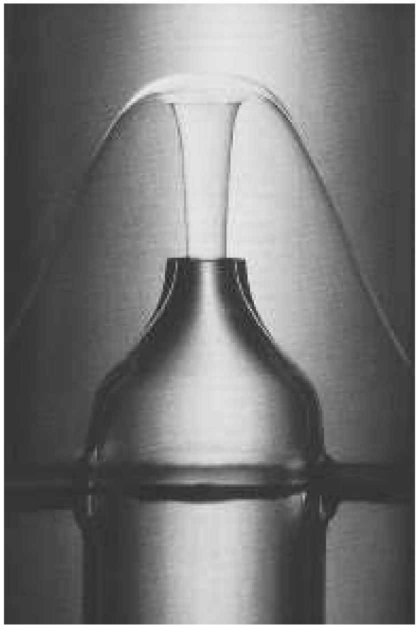
(Almost) scale invariant systems

Perfect Fluids: The contenders





QGP (T=180 MeV)



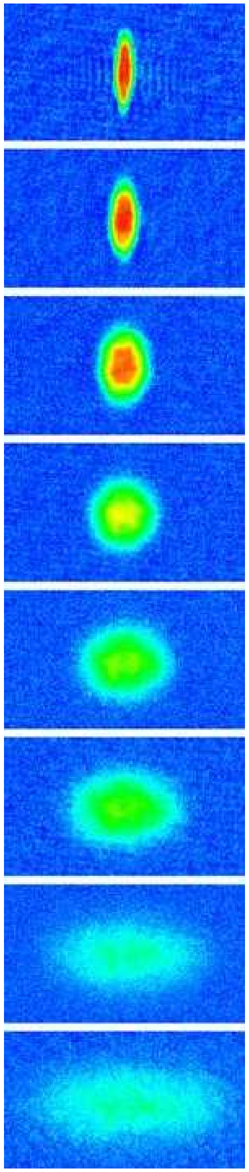
Liquid Helium

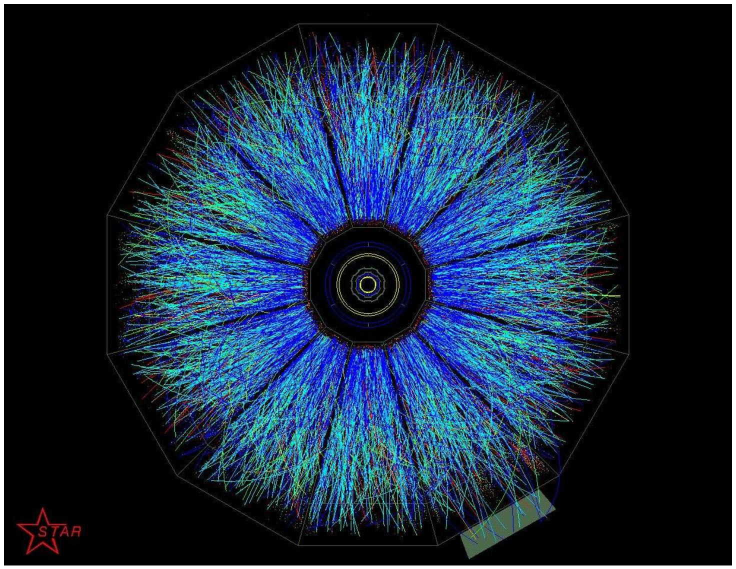
(T=0.1 meV)

Trapped Atoms

(T=0.1 neV)

Perfect Fluids: The contenders





QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$

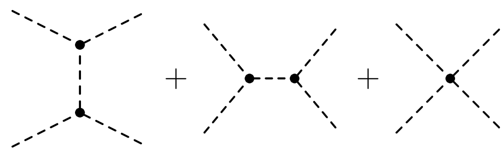
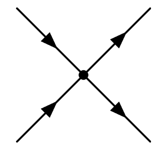
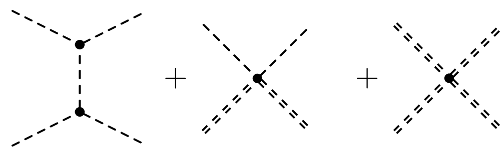
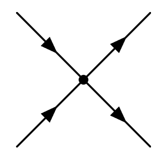
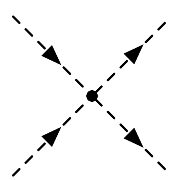
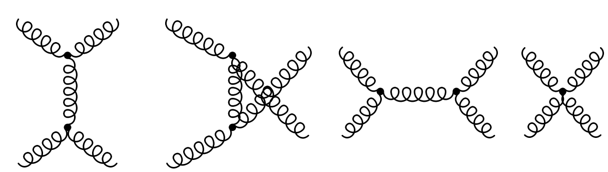
Liquid Helium

$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

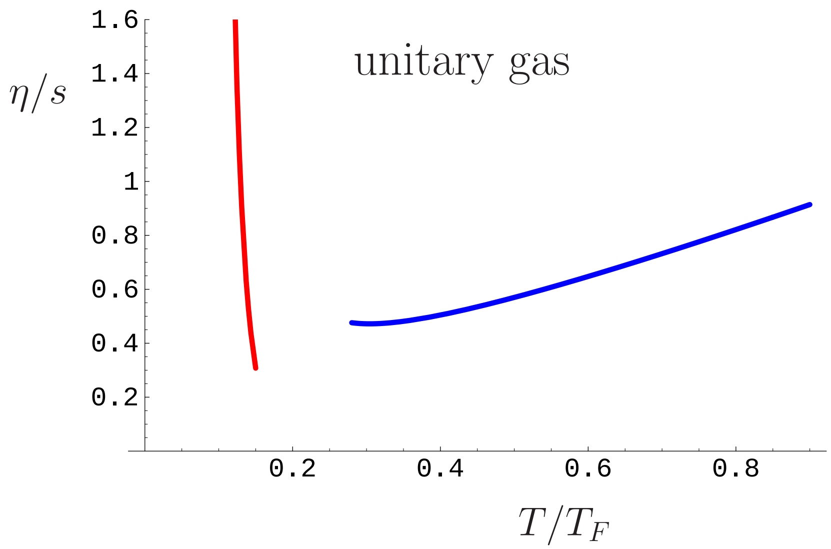
Consider ratios

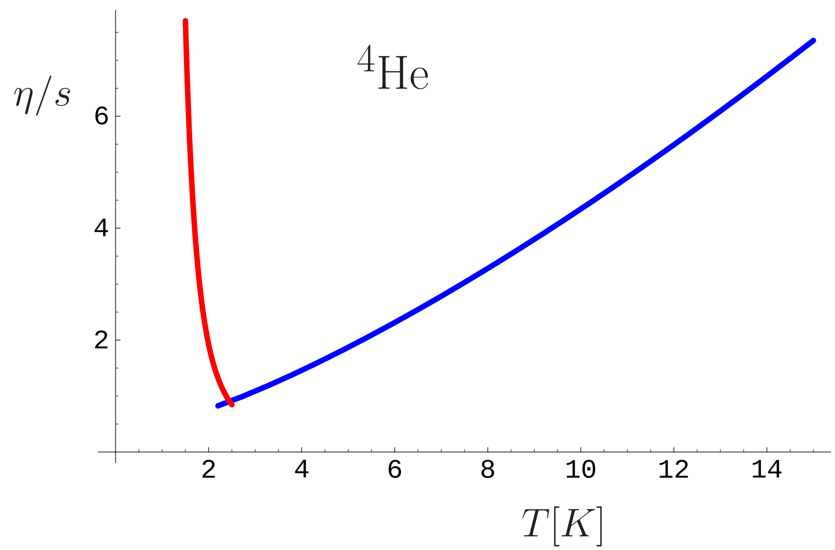
η / s

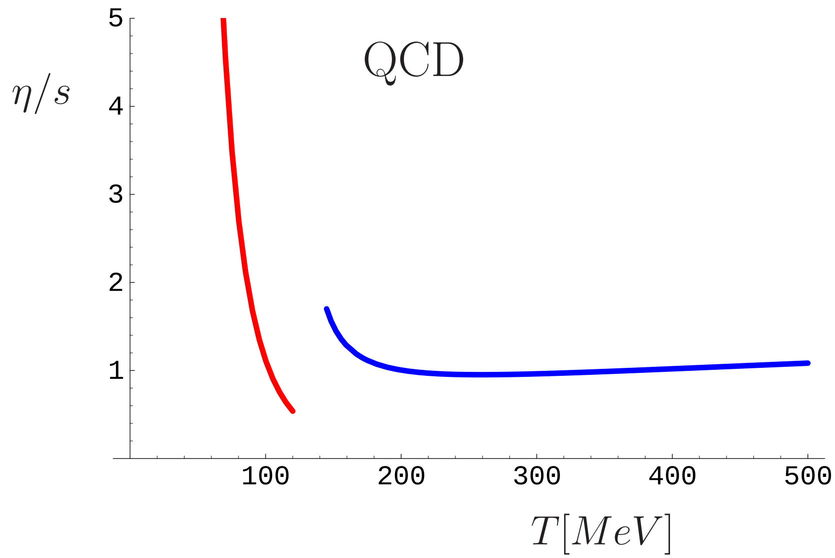
Kinetic Theory: Quasiparticles

	low temperature	high temperature
unitary gas	phonons	atoms
		
helium	phonons, rotons	atoms
		
QCD	pions	quarks, gluons
		

Theory Summary







I. Experiment (Liquid Helium)

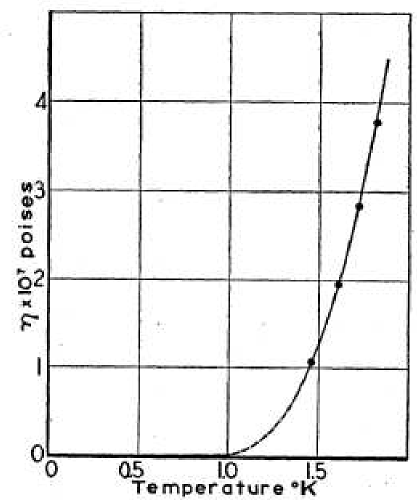
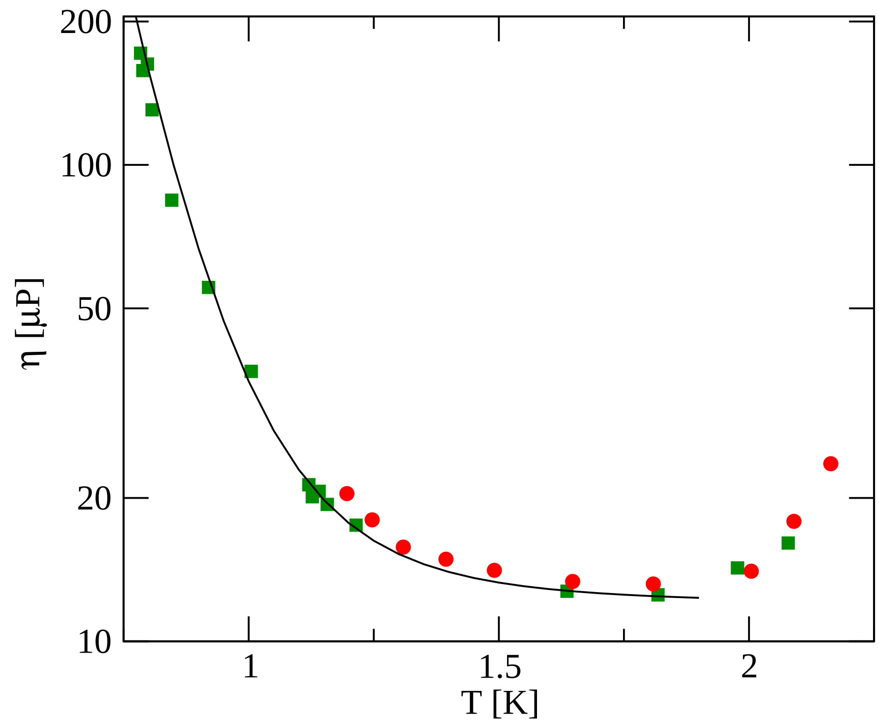


FIG. 1. The viscosity of liquid helium II measured by flow through a 10^{-4} cm channel.



Kapitza (1938)

viscosity vanishes below T_c

capillary flow viscometer

Hollis-Hallett (1955)

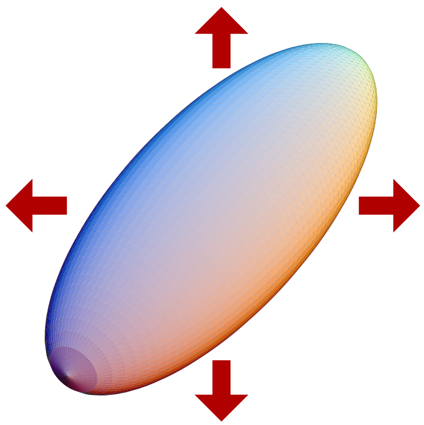
roton minimum, phonon rise

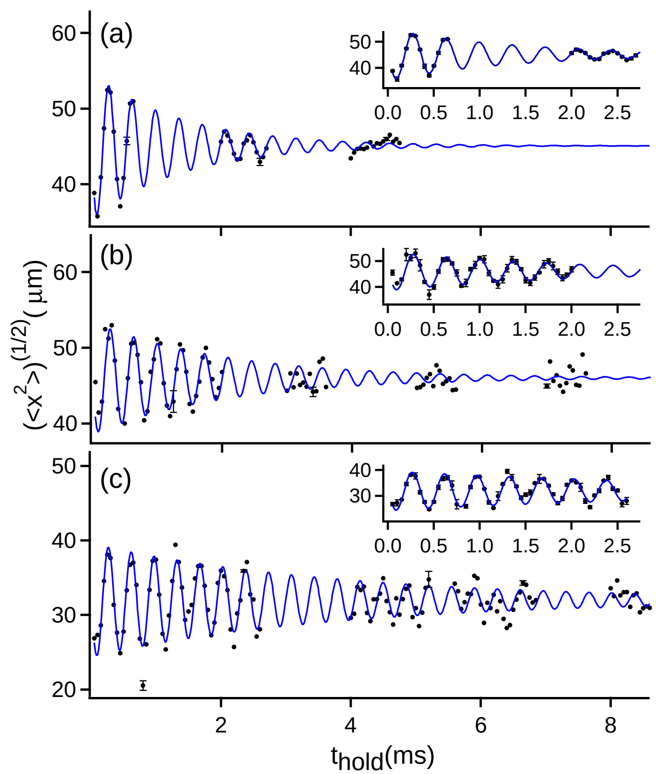
rotation viscometer

$$\eta/s \simeq 0.8\,\hbar/k_B$$

II. Hydrodynamics (Cold atoms)

Radial breathing mode





Ideal fluid hydrodynamics ($P \sim n^{5/3}$)

$$
\begin{aligned}
\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) &= 0 \\
\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla}\right) \vec{v} &= -\frac{\vec{\nabla} P}{mn} - \frac{\vec{\nabla} V}{m}
\end{aligned}
$$

Hydro frequency at unitarity

$$
\omega = \sqrt{\frac{10}{3}}\,\omega_\perp
$$

Damping small, depends on T/T_F.

experiment: Kinast et al. (2005)

Viscous Hydrodynamics

Energy dissipation (η, ζ, κ: shear, bulk viscosity, heat conductivity)

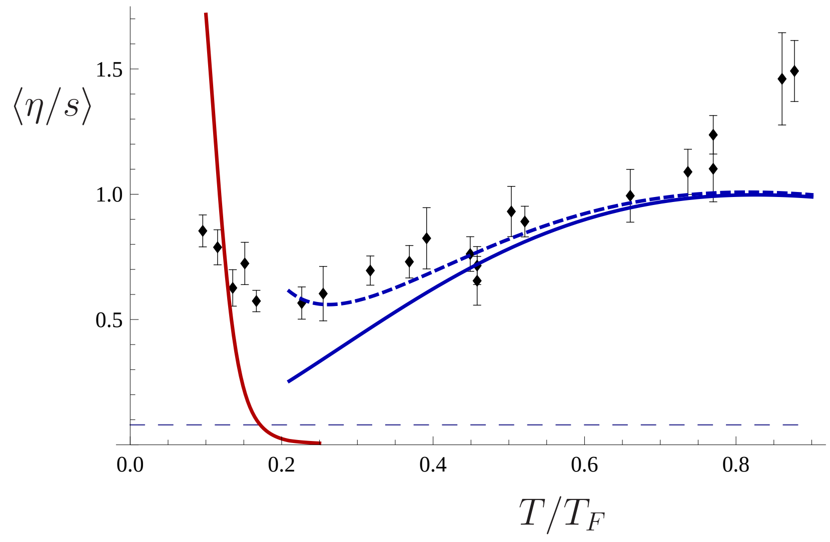
$$\dot{E} = -\frac{1}{2}\int d^3x\,\eta(x)\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 - \int d^3x\,\zeta(x)\,(\partial_i v_i)^2 - \frac{1}{T}\int d^3x\,\kappa(x)\,(\partial_i T)^2$$

Shear viscosity to entropy ratio

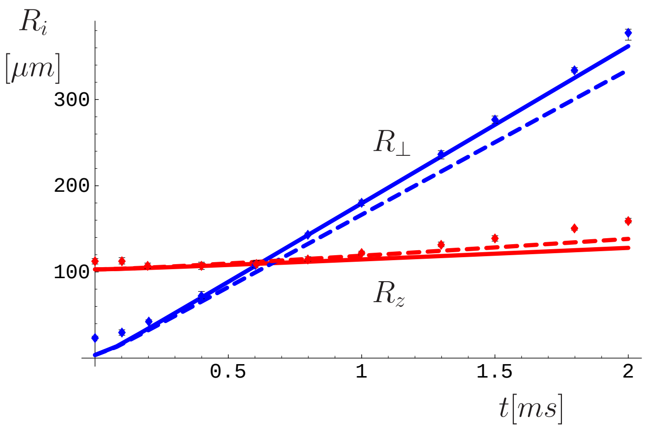
(assuming $\zeta = \kappa = 0$)

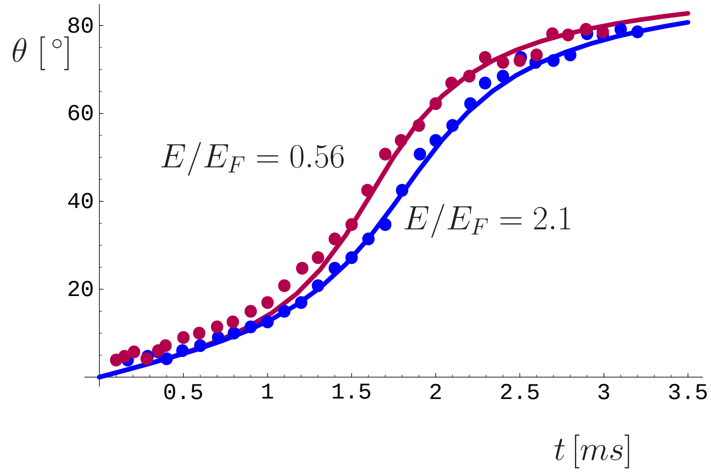
$$\frac{\eta}{s} = (3\lambda N)^{\frac{1}{3}}\,\frac{\Gamma}{\omega_\perp}\,\frac{E_0}{E_F}\,\frac{N}{S}$$

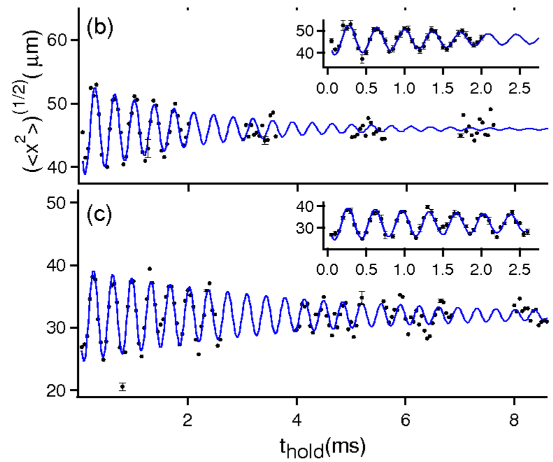
Schaefer (2007), see also Bruun, Smith

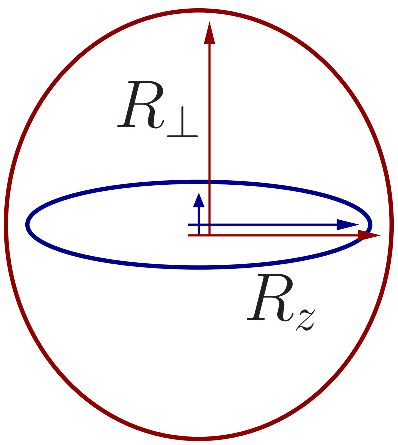


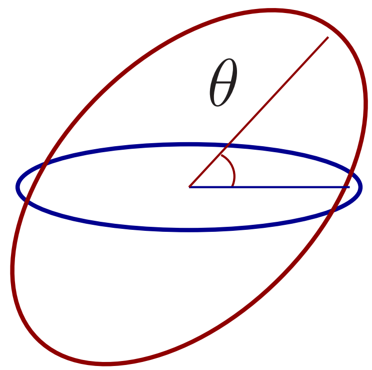
Dissipation

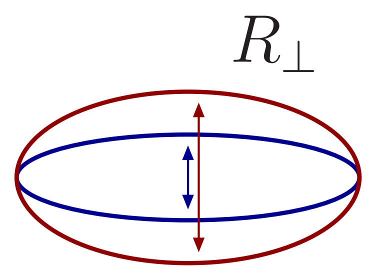






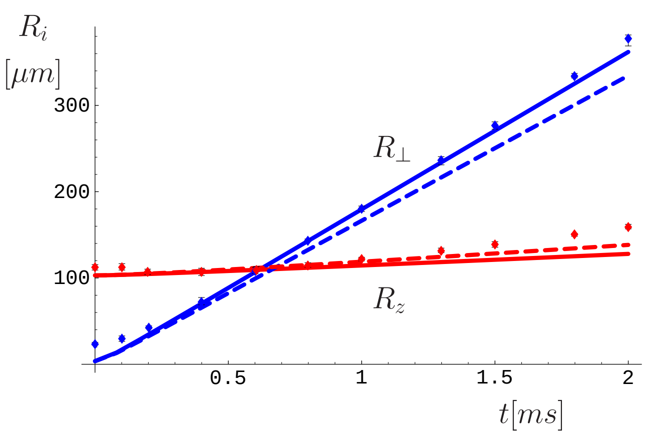


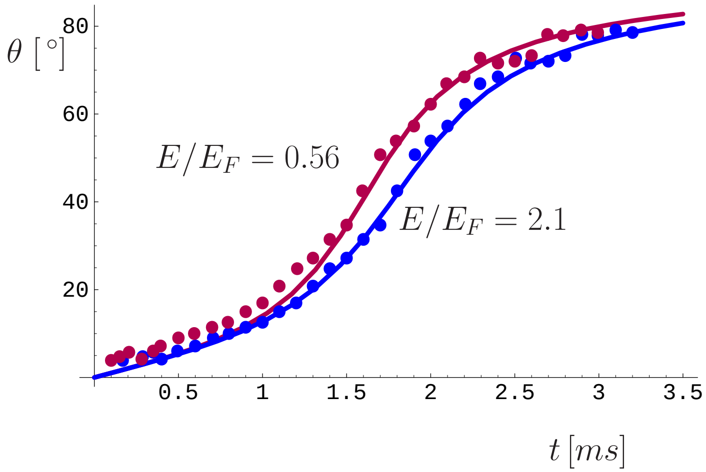


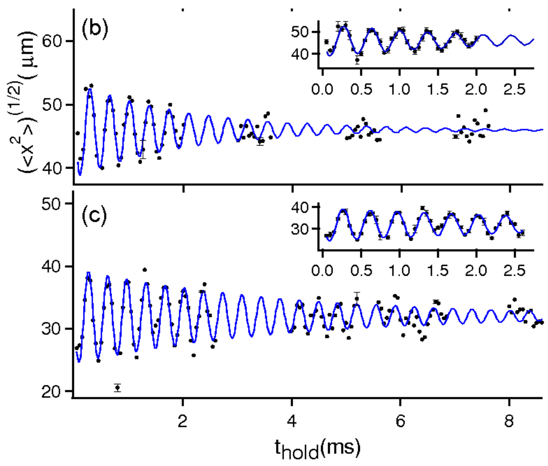


O'Hara et al (2002), Kinast et al (2005), Clancy et al (2007)

Dissipation







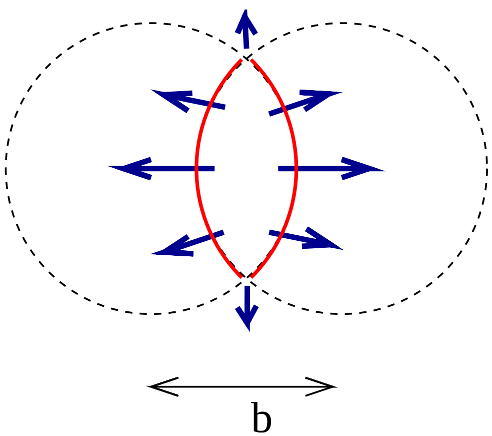
$$\left.\begin{array}{c}(\delta t_0)/t_0 \\ (\delta a)/a\end{array}\right\} = \left\{\begin{array}{c}0.008 \\ 0.024\end{array}\right\} \left(\frac{\langle\alpha_s\rangle}{1/(4\pi)}\right) \left(\frac{2\cdot 10^5}{N}\right)^{1/3} \left(\frac{S/N}{2.3}\right) \left(\frac{0.85}{E_0/E_F}\right)$$

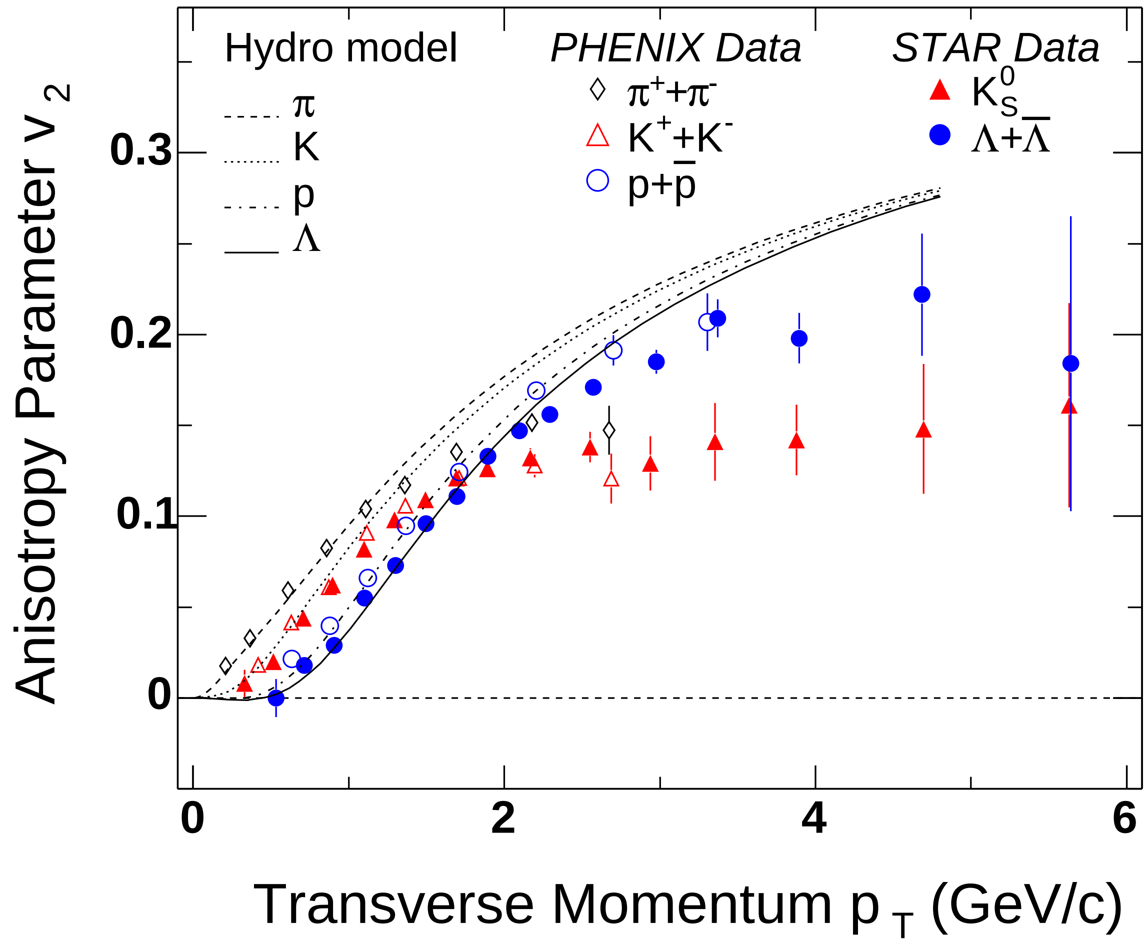
t_0: "Crossing time" $(b_\perp = b_z,\ \theta = 45^\circ)$

a: amplitude

III. Elliptic Flow (QGP)

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy





source: U. Heinz (2005)

Viscosity and Elliptic Flow

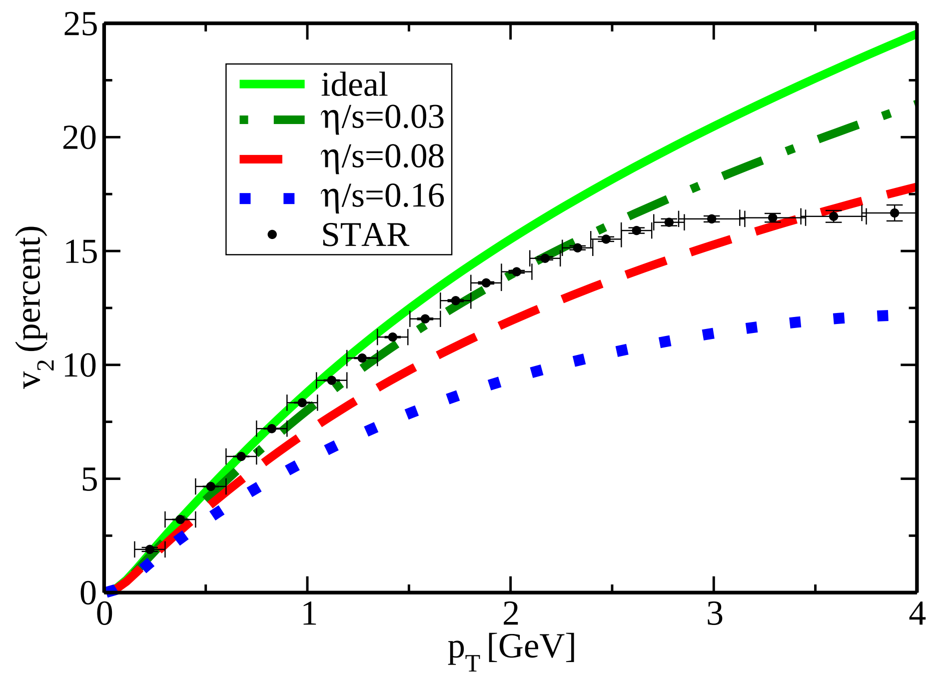
Consistency condition $T_{\mu\nu} \gg \delta T_{\mu\nu}$

(applicability of Navier-Stokes)

$$\frac{\eta + \frac{4}{3}\zeta}{s} \ll \frac{3}{4}(\tau T)$$

Danielewicz, Gyulassy (1985)

Very restrictive for $\tau < 1$ fm



Romatschke (2007), Teaney (2003)

Many questions: Dependence on initial conditions, freeze out, etc.

Outlook

Too early to declare a winner.

$$\eta/s \simeq 0.8 \text{ (He)}, \qquad \eta/s \leq 0.5 \text{ (CA)}, \qquad \eta/s \leq 0.5 \text{ (QGP)}$$

Other experimental constraints, more analysis needed.

Kinetic theory: o.k. in He (all T), o.k. close to T_c in CA, QGP?

New theory tools: AdS/Cold Atom correspondence? Field theory approaches in cross over regime (large N, epsilon expansions, ...)